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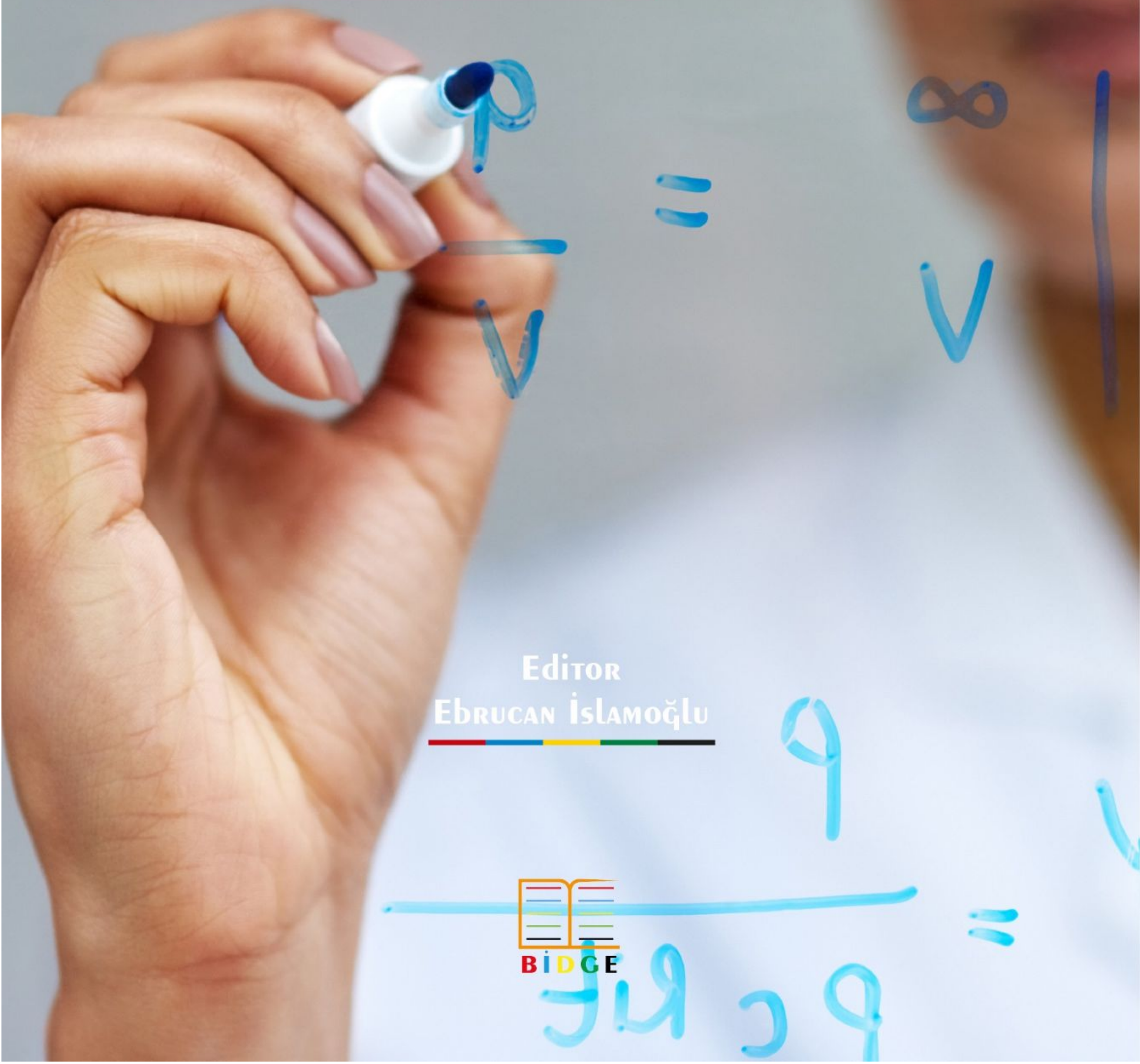

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# Content

# CHAPTER I

# On The Basic Structure of Logic $B_3$

**Hasan KELEŞ[1]**

## 1. Introduction

This chapter is first done on the enrichment of formal logic on a monoid. Here, new features, definitions, lemmas and propositions gained with the enrichment of logic are examined. Emphasis is placed on enriched structures. Additional developments in the usage areas of known structures are discussed. The connections of algebraic structures with power sets are examined. The main principle that algebraic structures add to the science of logic is discussed. The existence of new logical structures is investigated. The results of this new logical algorithm are evaluated. Comparisons are made with previously known algorithms. The structure is built on the set of solutions of some equations in the set

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of real numbers. The solution set of the equation $x^2 = x$ is composed of idempotent elements and the solution set of the equation $x^3 = x$ is composed of ternary elements. Boolean is formed binary logic from Arthur's idempotent elements. Binary logic is developed further today and created a digital field for itself. This widening of the field is also created some power problems. The problems are not initially physical problems. Different requirements are needed as the areas of use increased or as they are put into the service of humanity. This is contributed to the formation of triple logic. The material used by our mind in binary logic is two. This number is one more in ternary logic from binary logic. This situation is led to the acquisition of more skills in fewer operations. This structure is offered a solution for social structures. The use of the structure is likely to become widespread as humanity needs.

There are two important studies about logic written by Augustus De Morgan (1806-1871) and George Boolean (1815-1864), who are famous mathematicians of the 1847 period. These studies respectively are Formal Logic [4] and The Mathematical Analysis of Logic in [7]. The aim of the two authors is to develop a valid and consistent method and, as a result, to expand the theoretical logical structure with notations and transition to mechanical logic. De Morgan's letter to Boole explains this in 1847 [2]. Boole adopted the proposed logical methods by emphasizing the contrast of De Morgan in his study. De Morgan boldly acknowledged the originality of Boolean logical symbols. He started new studies on it and his study contributed to the enrichment of the structures in [1].

The set consisting of all subsets of a set $B$ is called the power set of $B$, written $\wp(B)$ in [7].

$$\wp(B)=\{X \mid X \subseteq B\}.$$

Let $B$ be a finite set of size $|B|=m$ Matheus Pereira Lobo is called the power set of the power set in [7, 9]. Non-empty sets are used in this study unless otherwise stated.

The set of all matrices of order n over a field $\mathbb{F}$ is denoted by $\mathbb{M}_n(\mathbb{F})$. Let us start with the definition below.

**Definition 1.1.** A group is a set $\mathbb{P}$ equipped with a binary operation $*:\mathbb{P}\times\mathbb{P}\rightarrow\mathbb{P}$ that associates an element $a*b\in\mathbb{P}$ to every pair of elements $a,b\in\mathbb{P}$, and having the following properties: $*$ is associative, has an identity element $e\in\mathbb{P}$, and every element in $\mathbb{P}$ is invertible. More explicitly, this means that the following equations hold for all $a,b,c,d\in\mathbb{P}$:

$P1$. $a*(b*c)=(a*b)*c$, (associativity).

$P2$. $a*e=e*a$, (identity).

$P3$. For every $a\in\mathbb{P}$, there is some $a^{-1}\in\mathbb{P}$ such that $a*a^{-1}=a^{-1}*a=e$ (inverse).

A set $\mathbb{P}$ together with an operation $*:\mathbb{P}\times\mathbb{P}\rightarrow\mathbb{P}$ and an element $e\in\mathbb{P}$ satisfying only conditions ($P1$) and ($P2$) is called a monoid in [3].

Noticed that the conditions for ($P1$) and ($P2$) are met on the multiplication operation in matrices. So let us briefly examine whether $\mathbb{M}_n(\mathbb{F})$ is a monoid or not. The set of $\mathbb{M}_n(\mathbb{F})$-square matrices satisfies the conditions ($P1$) and ($P2$) in [6], we know that for every regular matrices there are two matrices $B$ and $C$ with $B \neq C$ such that property $A = BA_1$, and $A = A_1C$

$P4$. For every $a \in \mathbb{P} \setminus \{e\}$, there are some $b, c, d \in \mathbb{P} \setminus \{e\}$ such that $\underbrace{b * \underbrace{c = c}_{=} * d}_{\neq} = a$ with $b \neq d$ (escort).

A set $\mathbb{P}$ together with an operation $* : \mathbb{P} \times \mathbb{P} \to \mathbb{P}$ and an element $e$ satisfying only conditions $P1$, $P2$, $P3$ and $P4$ is called a poloid. It is denoted by $(\mathbb{P}_p, *)$ in [6].

Here is a brief history of the monoid. The name "monoid" is first used in mathematics by Arthur Cayley for a surface of order which has a multiple point of order $n-1$. The elements $x \in \mathbb{P}$ satisfying the following equation are called the $k^{th}$ potent elements of the set $\mathbb{P}$:

$$\underbrace{x * x * \cdots * x}_{k-times} = x .$$

The set of all $k^{th}$ potent elements is denoted by $k^{(\mathbb{P}_p, *)}$ which is,

$$k^{(\mathbb{P}_p, *)} = \left\{ x \in \mathbb{P} \middle| \underbrace{x * x * \cdots * x}_{k-times} = x, k \in \mathbb{Z}^+ \setminus \{1\} \right\} = \wp(\mathbb{P}).$$

It is known that $\left| k^{(\mathbb{P}_p, *)} \right| = \left| \wp(\mathbb{P}) \right| = k^{|\mathbb{P}|} = k^m$.

If a set $\mathbb{P}$ is an infinite set, the set potent $k^{th}$ may be finite. For example, $k^{(\mathbb{R}, +)} = \{0\}$.

**Example 1.2.** Let $(\wp(B),\cup)$, then

$$k^{(\wp(B),\cup)}=\left\{X_i\in\wp(B)\left|\underbrace{X_i\cup X_i\cup\cdots\cup X_i}_{k-times}=x,k\in\mathbb{Z}^+\setminus\{1\},i=1,2,\ldots\right.\right\}$$

$$=\{X_1,X_2,\cdots\}.$$

**Example 1.3.** Let $(\wp(B),\cap)$, then

$$k^{(\wp(B),\cap)}=\left\{X_i\in\wp(B)\left|\underbrace{X_i\cap X_i\cap\cdots\cap X_i}_{k-times}=x,k\in\mathbb{Z}^+\setminus\{1\},i=1,2,\ldots\right.\right\}$$

$$=\{X_1,X_2,\cdots\}.$$

Now we give without proof, the following.

**Proposition 1.4.** Let $|\wp(\mathbb{P})|=\infty$ and $k\in\mathbb{Z}^+\setminus\{1\}$. Then,

i. $k^{(\wp(\mathbb{P}),\cap)}=k^{(\wp(\mathbb{P}),\cup)}$.
ii. $\left|k^{(\wp(\mathbb{P}),\cup)}\right|=\infty$.
iii. If $\mathbb{P}=\varnothing$ then $\left|k^{(\wp(\mathbb{P}),\cup)}\right|=1$.

The set of $k^{th}$ potent elements of $\mathbb{M}_n(\mathbb{F})$ is

$$k^{(\mathbb{M}_n(\mathbb{F}),.)}=\left\{X\in\mathbb{M}_n(\mathbb{F})\left|\underbrace{XX\ldots X}_{k-times}=X\right.\right\}.$$

## 2 The Basic Structure of Logic $B_3$

In this section, simple known algebraic structures are discussed. The defined concept of enrichment is analysed on these structures.

**Definition 2.1.** Let $(B,*)$ be a structure. If $\left|k^{(B,*)}\right| \geq 3$, then the structure $(B,*)$ is called a enriched structure of potent $k^{th}$. If $\left|k^{(B,*)}\right| = 1$, then $(B,*)$ is known a simplest structure. If $\left|k^{(B,*)}\right| = 2$, then $(B,*)$ is known a simple structure.

The structure of the basic structure of logic $B_3$ is the enriched structure in [8].

There is now a great deal of information about these simple and simplest structures. The enriched structure is sometimes due to defined the binary operation on the set, sometimes the binary operation adds enrichment to the set as an additional too in [5, 6].

**Example 2.2.** Let $(\mathbb{R},+)$. Then

$$k^{(\mathbb{R},+)} = \left\{ x \in \mathbb{R} \middle| \underbrace{x + x + \ldots + x}_{k-times} = x, k \in \mathbb{Z}^{+} \setminus \{1\} \right\}$$

If $k^{(B,+)} = 2$ then,

$$2^{(\mathbb{R},+)} = \left\{ x \in \mathbb{R} \middle| \underbrace{x + x}_{k-times} = x, k \in \mathbb{Z}^{+} \setminus \{1\} \right\} = \{0\}.$$

The structure $(\mathbb{R},+)$ is not an enriched structure. It is a simple structure.

**Example 2.3.** Let $(\mathbb{R},.)$. Then

$$k^{(\mathbb{R},.)} = \left\{ x \in \mathbb{R} \middle| \underbrace{x.x.\ldots.x}_{k-times} = x, k \in \mathbb{Z}^+ \setminus \{1\} \right\}$$

If $k^{(B,.)} = 2$ then,

$$2^{(\mathbb{R},.)} = \left\{ x \in \mathbb{R} \middle| x^2 = x, k \in \mathbb{Z}^+ \setminus \{1\} \right\} = \{0,1\}.$$

Also, if $k^{(B,.)} = 2$ then,

$$3^{(\mathbb{R},.)} = \left\{ x \in \mathbb{R} \middle| x^3 = x, k \in \mathbb{Z}^+ \setminus \{1\} \right\} = \{-1,0,1\}.$$

The structure $(\mathbb{R},.)$ is an only enriched structure of potent $3^{th}$ and it is the simple structure of potent $2^{nd}$.

It is clear that

$$2^{(\mathbb{R},.)} \subset 3^{(\mathbb{R},.)}.$$

**Proposition 2.4.** If $(\mathbb{P}_p,*)$ be a poloid. Then structure of the poloid $(\mathbb{P}_p,*)$ is a enriched structure.

**Proof.** Let $(\mathbb{P}_p,*)$ be a poloid. Suppose that $(\mathbb{P}_p,*)$ is not an enriched structure. Then

$\underbrace{x * x * \cdots * x}_{k-times} = x$ does not provide equality for any element of $(\mathbb{P}_p,*)$ other than unit.

Also,

$$\underbrace{x * x * \cdots * x}_{k-times} \neq x$$

by the definition of poloid.

$$\underbrace{x * x * \cdots * x}_{k-times} * x_2 \neq x * x_2 \text{, where } x_2 \in \mathbb{P}_p ,$$

$$\underbrace{x * x * \cdots * x}_{k-times} * x_3 \neq x * x_3 \text{, where } x_3 \in \mathbb{P}_p ,$$

Our assumption contradicts the definition of poloid. Thus, any poloid $(\mathbb{P}_p, *)$ is

an enriched structure. □

The reverse statement of this Proposition 2.4 is not true.

**Example 2.5.** The enriched structure $(\mathbb{R}, .)$ is not poloid.

**Example 2.6.** Let $(\mathbb{M}_n(\mathbb{C}), .)$. We have for any $\begin{bmatrix} x & y \\ z & t \end{bmatrix} \in \mathbb{M}_2(\mathbb{C})$

$$2^{(\mathbb{M}_n(\mathbb{C}),.)} = \left\{ \begin{bmatrix} i\sqrt{yz} & y \\ z & -i\sqrt{yz} \end{bmatrix}, \begin{bmatrix} -i\sqrt{yz} & x \\ y & i\sqrt{yz} \end{bmatrix} \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix} \right\}.$$

And $\left| 2^{(\mathbb{M}_n(\mathbb{C}),.)} \right| \geq 3$. Then $(\mathbb{M}_n(\mathbb{C}), .)$ is the enriched structure. The enrichment of the structure is not only depend to the power. It is also directly related with the structure.

**Lemma 2.7.** Let $(\mathbb{F},*,\circ)$ be a field. Then $2^{(\mathbb{F},*,\circ)}=\{0_{\mathbb{F}_\bullet},e_{\mathbb{F}_\circ}\}$.

**Proof.** For any $x\in\mathbb{F}$

$$x*x=x\Rightarrow x*\underbrace{x*x^{-1}}_{0_{\mathbb{F}_*}}=\underbrace{x*x^{-1}}_{0_{\mathbb{F}_*}}$$

$$\Rightarrow x=0_{\mathbb{F}_*}.$$

And,

$$x\circ x=x\Rightarrow x\bullet\underbrace{x\circ x^{-1}}_{e_{\mathbb{F}_\circ}}=\underbrace{x*x^{-1}}_{e_{\mathbb{F}_\circ}}$$

$$\Rightarrow x=e_{\mathbb{F}_\circ}.$$

Then

$$2^{(\mathbb{F},*,\circ)}=\{0_{\mathbb{F}_\bullet},e_{\mathbb{F}_\circ}\}\ \square$$

**Lemma 2.8.** Let $(\mathbb{F},*.\circ)$ be a field. If $k>2$ then

$$2^{(\mathbb{F},*,\circ)}\subseteq k^{(\mathbb{F},*,\circ)}.$$

**Proof.** Assump that $k=3$ for all $x\in\mathbb{F}\setminus\{0_{\mathbb{F}_*}\}$

$$x\circ x\circ x=x\Rightarrow x\circ x\circ\underbrace{x\circ x^{-1}}_{e_{\mathbb{F}_\circ}}=\underbrace{x\circ x^{-1}}_{e_{\mathbb{F}_\circ}}$$

$$\Rightarrow x\circ x=e_{\mathbb{F}_\circ}.$$

Here ${e^{-1}}_{\mathbb{F}_*}$ is the inverse element of the according to the operation $*$ of the $e_{\mathbb{F}_\circ}$ unit

element according to the operation $\circ$.

$$\Rightarrow (x \circ x) * {e^{-1}}_{\mathbb{F}_*} = e_{\mathbb{F}_\circ} * {e^{-1}}_{\mathbb{F}_*}$$

$$\Rightarrow \left(x * {e^{-1}}_{\mathbb{F}_*}\right) \circ \left(x * {e^{-1}}_{\mathbb{F}_*}\right)_{\mathbb{F}_*} = 0_{\mathbb{F}_*}$$

$$x * {e^{-1}}_{\mathbb{F}_*} = 0_{\mathbb{F}_*}$$

$$\Rightarrow x * \left({e^{-1}}_{\mathbb{F}_*} \circ e_{\mathbb{F}_\circ}\right) = e_{\mathbb{F}_\circ} * {e^{-1}}_{\mathbb{F}_*}$$

$$\Rightarrow \left(x * {e^{-1}}_{\mathbb{F}_*}\right) \circ \left(x * e_{\mathbb{F}_\circ}\right) = 0_{\mathbb{F}_*}$$

(1) If $x * {e^{-1}}_{\mathbb{F}_*} = 0_{\mathbb{F}_*}$ then $x = e_{\mathbb{F}_*} = e_{\mathbb{F}_\circ}$.

(2) If $x * e_{\mathbb{F}_*} = 0_{\mathbb{F}_*}$ then $x = {e^{-1}}_{\mathbb{F}_*}$.

Then,

$$3^{(\mathbb{F},*,\circ)} = \left\{{e^{-1}}_{\mathbb{F}_*}, 0_{\mathbb{F}_*}, e_{\mathbb{F}_\circ}\right\}.$$

This proof is easily completed by the induction method.for $k > 3$.

$$2^{(\mathbb{F},*,\circ)} \subseteq k^{(\mathbb{F},*,\circ)}.\square$$

**Example 2.9.** $k^{(\mathbb{M}_n(\mathbb{R}),+)} = \{[0]\}$. Because,

$$k^{(\mathbb{M}_n(\mathbb{R}),+)} = \left\{X \in \mathbb{M}_n(\mathbb{F}) \middle| \underbrace{X + X + \ldots + X}_{k-times} = X\right\}$$

$$=\left\{X \in \mathbb{M}_n(\mathbb{F}) \middle| kX = X\right\} = \{[0]\}$$

**Proposition 2.10.** Let $(\mathbb{F}, *. \circ)$ be any field. Then the following hold for any $x \in \mathbb{F} \setminus \{0_{\mathbb{F}_*}\}$ and $e_{\mathbb{F}_\circ} \in \mathbb{F}$.

i. $e^{-1}{}_{\mathbb{F}_*} \circ x = x \circ e^{-1}{}_{\mathbb{F}_*} = x^{-1}{}_{\mathbb{F}_*}$.

ii. $x^{-1}{}_{\mathbb{F}_*} \circ x = x \circ x^{-1}{}_{\mathbb{F}_*} = (x \circ x)^{-1}{}_{\mathbb{F}_*}$.

**Proof.** Let $(\mathbb{F}, *. \circ)$ be any field. There is only one unit element $e_{\mathbb{F}_\circ} \in \mathbb{F}$ according to the operation $\circ$.

i. We just need to prove the equality $e^{-1}{}_{\mathbb{F}_*} \circ x = x^{-1}{}_{\mathbb{F}_*}$.

$$e_{\mathbb{F}_\circ} * e^{-1}{}_{\mathbb{F}_*} = 0_{\mathbb{F}_*}$$

$$\left(e_{\mathbb{F}_\circ} * e^{-1}{}_{\mathbb{F}_*}\right) \circ x = 0_{\mathbb{F}_*} \circ x = 0_{\mathbb{F}_*}$$

$$\underbrace{\left(e_{\mathbb{F}_\circ} \circ x\right)}_{x} * \left(e^{-1}{}_{\mathbb{F}_*} \circ x\right) = 0_{\mathbb{F}_*}$$

$$x * \left(e^{-1}{}_{\mathbb{F}_*} \circ x\right) = 0_{\mathbb{F}_*} \Rightarrow e^{-1}{}_{\mathbb{F}_*} \circ x = x^{-1}_{\mathbb{F}_*}.$$

ii. Now we prove the equality $x^{-1}{}_{\mathbb{F}_*} \circ x = (x \circ x)^{-1}{}_{\mathbb{F}_*}$

$$x^{-1}{}_{\mathbb{F}_*} \circ x * (x \circ x) = \left(\underbrace{x^{-1}{}_{\mathbb{F}_*} * x}_{0_{\mathbb{F}_*}}\right) \circ x = 0_{\mathbb{F}_*} \circ x = 0_{\mathbb{F}_*}$$

$$x^{-1}_{\mathbb{F}_*} \circ x = (x \circ x)^{-1}{}_{\mathbb{F}_*}. \square$$

**Lemma 2.11.** Let $(\mathbb{F},*.\circ)$ be any field. Then

$$(2k)^{(\mathbb{F},*,\circ)} \subseteq (2(k+1))^{(\mathbb{F},*,\circ)} \text{ , where } k \geq 1 .$$

**Proof.** For any $x \in (2k)^{(\mathbb{F},*,\circ)}$

$$\underbrace{x * x * \cdots * x}_{2k-times} = x$$

$$\underbrace{x * x * \cdots * x}_{2k-times} = \underbrace{x * x * \cdots * x}_{(2k-2)-times} * \underbrace{x}_{x^2} * \underbrace{x}_{x^2}$$

$$\underbrace{x * x * \cdots * x}_{2k-times} = \underbrace{x * x * \cdots * x}_{(2k-2)-times} * x^2 * x^2 = \underbrace{x * x * \cdots * x}_{(2k+2)-times} = x$$

$$\Rightarrow x \in (2(k+1))^{(\mathbb{F},*,\circ)} .\square$$

**Theorem 2.12.** Let $(\mathbb{F},*.\circ)$ be any ring or field. Then $(\mathbb{F},*.\circ)$ is a enriched structure of potent $k^{th}$, where $k \geq 2$ .

**Proof.** If $(\mathbb{F},*.\circ)$ is any ring or field, then $(\mathbb{F},*.\circ)$ is the enriched structure and $\left|k^{(\mathbb{F},*.\circ)}\right| \geq 3$ by Lemma 2.8. □

**Theorem 2.13.** For any $k \in \mathbb{Z}^{+} \setminus \{1\}$ the following statements hold.

i. $(2k+1)^{(\mathbb{R},.)} = 3^{(\mathbb{R},.)}$ .

ii. $(2k)^{(\mathbb{R},.)} \subset 3^{(\mathbb{R},.)}$ .

**Proof.**

i.

(A1) For any $x \in (2k+1)^{(\mathbb{F},*,\circ)}$

$$x^{2k+1} = \underbrace{x^{2k}}_{x^2} x = x^3 = x \Rightarrow x \in 3^{(\mathbb{R},.)}$$

$$\Rightarrow (2k+1)^{(\mathbb{R},.)} \subset 3^{(\mathbb{R},.)}$$

(A2) For any $x \in 3^{(\mathbb{F},*,\circ)}$

$$x^3 = \underbrace{x^2}_{x^{2k}} x = x^{2k} x = x^{2k+1} = x \Rightarrow x \in (2k+1)^{(\mathbb{R},.)}$$

$$\Rightarrow 3^{(\mathbb{R},.)} \subset (2k+1)^{(\mathbb{R},.)}.$$

$$(2k+1)^{(\mathbb{R},.)} = 3^{(\mathbb{R},.)}$$

by A.(1) and A.(2) .

ii. Let us prove with the induction method. The statement $k=1$ says that

$$(2)^{(\mathbb{R},.)} \subset 3^{(\mathbb{R},.)}$$

It is true. Suppose that $k-1$ holds, that is,

$$(2(k-1))^{(\mathbb{R},.)} \subset 3^{(\mathbb{R},.)}.$$

It remains to show that k holds, that is,

$$x^{2k} = \underbrace{x^{2(k-1)}}_{x^3} x^2 = x$$

$$x^{2(k-1)} = x^3 = x$$

$$x^{2(k-1)} = x^3 = x$$

by Theorem 2.13 (i). Then

$$x^{2k} = \underbrace{x^{2(k-1)}}_{x} x^2 = x^3 = x \Rightarrow x \in 3^{(\mathbb{R},.)}$$

$$(2k)^{(\mathbb{R},.)} \subset 3^{(\mathbb{R},.)}.$$

Real numbers are sufficient for most of equations. The solutions of equations $x^2 = x$ and $x^3 = x$ are real numbers. But the roots (or zeros) of the following equations are complex numbers as well.

$$z^2 = z \text{ and } z^3 = z$$

Thus, the sets of idempotent elements for any $z \in \mathbb{C}$ are:

$$2^{(\mathbb{C},+)} = \left\{ z \in \mathbb{R} \middle| z + z = z, k \in \mathbb{Z}^+ \setminus \{1\} \right\} = \{0\}$$

If $x = 0$ and $x = 1$ for any complex number $z = x + iy$, where $x, y \in \mathbb{R}$, then,

$$2^{(\mathbb{C},.)} = \left\{ z \in \mathbb{R} \middle| z^2 = z, k \in \mathbb{Z}^+ \setminus \{1\} \right\} = \{0,1\} = 2^{(\mathbb{R},.)}.$$

**Theorem 2.14.** For any complex number $z = x + iy$ the following hold.

i. If $x \in (-\infty, 0] \cup [1, +\infty)$ then,

$$2^{(\mathbb{C},.)} = \left\{ x \pm i\sqrt{x^2 - x} \right\}.$$

ii. If $x \in (-\infty,-1] \cup [1,+\infty) \cup \left(-\infty, \frac{-\sqrt{3}}{3}\right] \cup \left[\frac{\sqrt{3}}{3}, +\infty\right)$ then,

$$3^{(\mathbb{C},.)} = \left\{ x-\sqrt{3x^2-1}, x+\sqrt{3x^2-1} x-\frac{\sqrt{3}}{3}\sqrt{x^2-1}, x-\frac{\sqrt{3}}{3}\sqrt{x^2-1}, \right\}$$

.

**Proof.**

i. From the solution of the equation $z^2 = z$, if $x = \frac{1}{2}$ then $y = \mp\sqrt{x^2 - x}$ is obtained.

Therefore,

$$2^{(\mathbb{C},.)} = \left\{ x \pm i\sqrt{x^2 - x} \right\}$$

Similarly,

ii. The equations $y = \pm\frac{\sqrt{3}}{3}\sqrt{x^2-1}$ and $y = \pm\sqrt{3x^2-1}$ are obtained from

the solution of the equation $z^3 = z$. And,

$$3^{(\mathbb{C},.)} = \left\{ x-\sqrt{3x^2-1}, x+\sqrt{3x^2-1} x-\frac{\sqrt{3}}{3}\sqrt{x^2-1}, x-\frac{\sqrt{3}}{3}\sqrt{x^2-1}, \right\}$$

.□

## 3. Conclusion

The concept of "poloid", which has just been defined as our knowladge, will find many application areas. The existence of an algebraic structure that manifests itself when any element is processed from the right and left is still an open problem. The reconsideration of the relationship between known algebraic structures and logic will lead to new discussions with the examination of its basic structures.

Algebraic structures constitute the basic structure of computer science. It is clear that this will lead to the development of the concept of logic created by structures. In short, algebraic richness adds new expressions to the "exist-no" pair in logic is open to discussion.

## References

Barnett, H. J., (2016). Origins of Boolean Algebra in the Logic of Classes, Mathematical Association of America-Pueblo: Colorado State University.

Boehm, J., (2016). Grundlagen der Algebra und Zahlentheorie, Berlin, Heidelberg: SpringerLehrbuch. Springer Spektrum, 2016.

Corry, L., (1996). *Modern Algebra and the Rise of Mathematical Structures*, Basel, Boston, Berlin: Birkhauser Verlag.

Gallier, J. and Quaintance, J., (2022). Algebra, Topology, Differential Calculus, and Optimization Theory for Computer Science and Machine Learning, Book in Progress, Approx.

Keilen, T., (2016). Algebra I, Mathematics Institute: University of Warwickr.

Keleş, H., (2022). Poloids and Matrices, The Aligarh Bulletin of Mathematics,(4) **1**, 41-52.

Lobo, M. P., (2013). The Coalgebraic Enrichment of Algebras in Higher Categories, Open Journal of Mathematics and Physics:230.

Proux, M., (2022). The Coalgebraic Enrichment of Algebras in Higher Categories, Journal of Pure and Applied Algebra, 226:106849.

Zach, R., (2017). Sets Logic Computation An Open Introduction to Metalogic, University of Calgary: Matthew Leadbeater.

# CHAPTER II

# An Analysis of Some Notable Blockchain-based E-auction Systems

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## 1. Introduction

An electronic auction, often known as an e-auction, allows selling items via auctioning over the internet. E-auction has a mobility advantage over traditional auctions since it eliminates the physical presence of the buyer and seller at a central location (Trevathan, 2005). Thus, many companies (such as eBay) have moved to e-auction systems. Typically, e-auction systems contain three types of participants: seller, bidder, and auctioneer (Braghin,

Cimato and Damiani, 2020). While seller, also known as the vendor, is willing to sell an item, the bidder submits a price to buy the item.

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Also, the auctioneer manages the auction according to the predefined rules, thereby promoting the bidder's trust.

In general, auctions are classified into four types based on the openness of the auction price, and the determination of the winning rule: open ascending price auctions (English), open descending price auctions (Dutch), first-price sealed-bid auctions, and second-price sealed-bid auctions (Alvarez and Nojoumian, 2020). While open ascending price auctions require increasing public open bids, open descending price auctions demand decreasing public open bids. On the other hand, while first-price sealed-bid auctions define the winning bid price as the highest bid value, second-price sealed-bid auctions define it as the second-highest bid. In sealed-bid auctions, only the winning bid price is revealed, thus protecting the bidder's anonymity and the bid's privacy through some cryptographic primitives (Li and Xue, 2021). Preventing information leakage makes sealed-bid auctions desirable for both bidders and sellers. However, to create a secure sealed-bid e-auction system, the seller and bidder must engage a third-party intermediary to trade the auction (Chen, Chen and Lin, 2018). This intermediary assists with tasks such as product posting, monitoring the highest bidding price, and finalizing the transaction with the winning bidder. Fortunately, blockchain technology eliminates the need for trust in centralized third-party intermediaries.

In the context of an e-auction, blockchain technology has many advantages due to its decentralized structure. The core technology of blockchain guarantees that the auction is transparent, tamper-proof, unforgeable, and traceable (Ye et al., 2023). Blockchain technology can be used in two ways (Germouty, Larraia and Zhang, 2023). Firstly, blockchain can serve as a bulletin board where data is published by bidders and the auctioneer, and with the data integrity provided in blockchain, the data can be verified by any third party. Secondly, blockchain can function as a verification tool, enabling a self-enforced, transparent auction. Specifically, the verification process can be automated via smart contract by defining the deadline as a function of the number of bids, block length, or time (Gysel, Ford and Zhang, 2023). In sum, the integrity of recorded data and transparent verification can come to the rescue of trust assumptions in centralized intermediaries, ensuring the accuracy of the auction results. However, it is important to acknowledge that the use of blockchain technology in sealed-bid e-auctions necessitates the incorporation of additional cryptographic primitives, such as commit-reveal, to keep the bid secret from any other bidders.

The advantages of blockchain in sealed-bid e-auction systems were also evaluated in recent studies (Shi et al., 2022) (Khandelwal et al., 2022) (Liu et al., 2024). The primary goal of this study is to evaluate some prominent blockchain-based sealed-bid e-auction systems in terms of their achieved requirements and drawbacks. Based on this, we provide potential cryptographic primitives that may be used to fulfill the requirements of an e-auction system.

The rest of the paper is organized as follows: Section 2 provides information about the stages and the design of an e-auction system as preliminaries. Section 3 analyzes some notable blockchain-based sealed-bid auction systems based on their achieved design requirements and drawbacks. Section 4 presents the conclusion.

## 2.Methodology

### 2.1. Stages of E-Auction Systems

General stages of e-auction systems for proper management of the auction can be categorized as follows (Alvarez and Nojoumian, 2020) (Chen, Chen and Lin, 2018):

- Participant Registration: In order to participate in an auction, bidders must register with the system via a dedicated tool or a third-party.
- Bid Acceptance: Once the product description is posted by the seller, bidders may submit their bid prices to the auctioneer, either publicly or in a secret manner.
- Bid Opening: Comparing all bid prices, the auctioneer announces the winning bid price once the deadline is due. Subsequently, the auctioneer takes the money from the winner and sends the product to that bidder.

### 2.2. Design Goals of E-Auction Systems

E-auction systems have some design goals to prevent potential threats that may compromise the auction system. An ideal e-auction system should achieve some fundamental system properties to establish the trust of the user (Alvarez and Nojoumian, 2020). In this context, comprehensive definitions of these properties can be itemized as follows (Dreier, Lafourcade and Lakhnech, 2013):

- Secrecy of bids: It ensures the losing bid remains hidden, or at least it cannot be traceable to the participants.
- Anonymity of bidders: It guarantees that the participants remain anonymous.
- Fairness: It means there is no intermediate information about the participants and their bids before the auction is over.
- Non-repudiation: It is the winning bidder's failure to demonstrate that the winning bid was not sent by him. It is referred to as bid-binding (Germouty, Larraia and Zhang, 2023).
- Robustness: It means it is impossible to disrupt the system despite the malicious behavior of parties.
- Verifiability: It can be defined in two stages: registration and integrity verifiability (RV), and outcome verifiability (OV) (Dreier, Lafourcade and Lakhnech, 2013). In the first one, anyone can verify that all published bids are submitted by registered bidders and that the winning bid is one of them. The second one is run from three different perspectives: the losing bidder that verifies his bid was not the winning bid, the winning bidder that verifies his bid was not the winning bid, and the seller that verifies the highest bid is the winning bid.

## 3.Some Notable Blockchain-based E-auction Systems and Their Comparisons

Blass and Kerschbaum (2018) proposed Strain (Secure aucTion foR blockchAINs), a sealed-bid auction protocol. In this protocol, bidders encrypt their committed bids with their public key using extended Goldwasser-Micali encryption (Goldwasser and Micali, 2019). This key is produced through a distributed key generation method, ensuring that an honest majority can decrypt the ciphertext in case of a conflict. Thus, it offers a reversible commitment system. In addition to this encryption, bidders publish the related zero-knowledge proofs for the accuracy of the outcome on the blockchain. Once the auction closes, a two-party comparison process determines the result by simultaneously comparing any pair of bids and a public outcome. However, as stated, this protocol leaks the order of bids.

Desai et al. (2019) proposed a hybrid blockchain-based auction system that uses a private blockchain to open secret bids only by the auctioneer and a public blockchain to make the auction winner accountable, which is a strong form of verifiability. In the system, bidders generate a hash value on their bids using standard bit commitments, and the public blockchain stores this value. When the auction ends, each bidder sends opening commitment values to the auctioneer through the auction smart contract (in the private blockchain). After checking the consistency, an auctioneer declares the winner through PublicDeclare smart contract (in the public blockchain). In the case of the malicious auctioneer, bidders can check the values used by the auctioneer and their commitments through the CongressFactory contract, which forces the auctioneer to send the bids. This system preserves the user's privacy while providing accountability, but the role of the auctioneer in the private chain weakens the security compared to a trustworthy smart contract, and the public portion of the smart contract introduces scalability problems (Keizer et al., 2021).

Galal and Youssef (2019) proposed an Ethereum based e-auction scheme that uses smart contract to take additively homomorphic bid commitments and opening values as transactions, then uses off-chain winner determination by the auctioneer to keep the secrecy of losing bids from the transparent blockchain. According to this scheme, bidders submit commitments on their sealed bids with their zero-knowledge proof of interval membership to the smart contract. After the auction process, bidders disclose their commitments, and the auctioneer computes the differences between each pair of bids through the property of an additively homomorphic commitment scheme. Finally, the auctioneer publishes the winner and provides comparison zero-knowledge proofs of the correctness of the winner to contract. Nevertheless, the auctioneer in this system has the capability to compromise bid privacy.

Li and Xue (2021) proposed an e-auction scheme with smart contracts to remove third-party auctioneers, commitments to protect bid information, and zero-knowledge proof to verify auction results by all anonymous bidders. In this scheme, all bidders publish their committed bids on the blockchain. Also, plain bid and the random blinding factor used in commitment are encrypted using the smart contract public key and sent to the smart contract. When the auction is closed, the smart contract opens commitments, then reveals the winning commitment that is bound to the winning bid and sends the differential commitments to all bidders. Each bidder uses differential commitments and generates bulletproofs to prove the revealed winning bid is higher than his bid value. The smart contract announces the successful completion when it receives all proofs. However, the running time of open and finish operations significantly increases with the number of bidders. Also, the scheme gives a user ID and key pair to each bidder as registration data but doesn't consider that the adversary can corrupt the auction by getting the identity of the bidder (Zhang, Yang and Shen, 2022).

Zhang et al. (2022) use time-lock encryption to prevent the case that a malicious auctioneer sells the bid information obtained before the deadline to malicious bidders, thus guaranteeing bid security. In addition, they use smart contracts for auction verification to avoid malicious auctioneer cases. This system integrates the time-lock puzzle into the block difficulty to regulate the corresponding block generation time, which publishes a private key corresponding to encrypted bids. Consequently, a time-released blockchain prevents the decryption of bid information during the auction. Also, a smart contract is used to transparently decrypt these bids. However, this scheme returns registration identities to bidders through the auctioneer, thus relying on him.

Ye et al. (2023) provide an auction mechanism that employs a Schnorr signature to enhance system efficiency, a ring signature to ensure bidder anonymity, a blockchain for transparency, and an interplatenary file system (IPFS) network to address storage challenges in the blockchain. In this system, sellers submit the auction information to IPFS as a hash value, then sign the encrypted hash value using Schnorr signature and transmit it to the online auction platform (OAP) along with a timestamp value. The OAP verifies the timestamp and signature before making adjustments to the ledger. During the auction phase, the bidder utilizes a ring signature to authenticate and transmit its price value to OAP. The OAP updates the ledger and then announces the winner. The dispute arbitrator is responsible for addressing any irregularities in the deposit contract and the signing of the winning offer. However, in this mechanism, the user's key pairs are generated by OAP and then sent to the user; thus, their confidential transmission is an important issue.

Chin et al. (2024) proposed a blockchain-based sealed bid auction protocol that focuses on fund-binding property, which ensures that a bidder has at least his offered bid price as a fund, thus overcoming false bidding scenarios. While most common protocols overcome this vulnerability by using a simple deposit method in which each bidder sends a deposit to the smart contract that is more than or equal to his bid price before bidding, this ruins bid secrecy, revealing the maximum price. Chin et al. (2024) suggested using a decentralized oracle (DECO) that sends data (such as ETH price) from outside the blockchain to the smart contract and uses one-time addresses for the fund-binding contract. An auctioneer in the system uses an auction smart contract to collect secret bids and determine the winner. Each bidder transfers funds to the fund-binding contract's one-time address, deployed from the smart contract, using a unique address. Next, the bidder submits a bid price hash-based commitment into the auction contract and uses DECO to prove the one-time address has enough balance using a three-party handshake in TLS. Lastly, the auction contract deploys a fund-binding contract and withdraws the funds from these addresses. In this case, the bidder is the only person who knows the one-time address with a special salt value; thus, each bidder needs to reveal salt for the auction contract to withdraw the fund.

Considering these notable blockchain-based sealed-bid e-auction systems, Table 1 analyzes their focused system requirements and drawbacks.

***Table 1**: Comparison of some notable blockchain-based sealed-bid auctions*

| Scheme | Year | Underlying Primitive | Focused Requirements | Drawbacks |
|---|---|---|---|---|
| Blass and Kerschbaum | 2018 | Two-party comparison proof, Reversible commitment with threshold key generation | Secrecy of bid, Fairness, Robustness against dispute, Fairness, Public verifiability | Leaks the order of bids |
| Desai et al. | 2019 | Hybrid blockchain, Smart contract, Commitment | Secrecy of bid, Fairness, Public verifiability | Scalability problems in public smart contract |
| Galal and Youssef | 2019 | Smart contract, Zero-knowledge proof, Pdersen commitment | Secrecy of bid, Fairness, Public verifiability | Need an auctioneer for verification |

| | | | | |
|---|---|---|---|---|
| Li and Xue | 2021 | Smart contract, Bulletproofs, Pedersen commitment | Secrecy of bid, Fairness, Public verifiability | Time-consuming operations, Authentication threats |
| Zhang et al. | 2022 | Smart contract, Time-lock encryption | Secrecy of bid, Fairness | Authentication threats |
| Ye et al. | 2023 | Schnorr signature, Ring signature, Interplatenary File System | Anonymity of bidders, Fairness Non-repudiation | Require confidential transmission of bidder's key pair |
| Chin et al. | 2024 | Smart contract, Decentralized oracle with one-time address, Hash-based commitment | Secrecy of bid, Anonymity of bidders, Robustness against dispute | Require bidder to reveal their private salt correctly |

Based on Table 1, it is important to note that certain cryptographic primitives fulfill an e-auction system's design requirement. The relation between them is seen in Table 2. According to this table, while proper usage of commitments and time-lock encryption achieves both secrecy of bids and fairness, standard public key encryption algorithms offer only secrecy of bids. Bidders can achieve anonymity through one-time blockchain addresses and ring signatures, while standard signature algorithms ensure non-repudiation. Additionally, threshold cryptography, which includes threshold encryption, threshold key generation, and the blockchain platform, can provide robustness. Lastly, the use of zero-knowledge proofs, blockchain platforms, and smart contracts may ensure the verifiability of the election results.

***Table 2:*** *Relation between the design requirement of an e-auction system and cryptographic primitive*

| Requirements | Cryptographic Primitive |
|---|---|
| Secrecy of bids | Commitment, Standard Public Key Encryption, Time-lock Encryption |
| Anonymity of bidders | One-time blockchain address, Ring signature |
| Fairness | Commitment, Time-lock Encryption |
| Non-repudiation | Standard Signature |
| Robustness | Threshold Cryptography (Encryption, Key Generation), Blockchain Platform |
| Verifiability | Interactive / Non-interactive Zero-knowledge proof, Blockchain Platform, Smart Contract |

## 4. Conclusion

Since blockchain technology is gaining prominence in e-auction systems with its decentralized and tamper-proof structures, these advantages have also been evaluated in e-auction systems. This work analyzed some notable blockchain-based sealed-bid auction schemes in terms of their design requirements and drawbacks. Subsequently, we described the relationship between the design requirements and their potential cryptographic primitives.

## References

Alvarez, R., & Nojoumian, M. (2020, January 1). Comprehensive survey on privacy-preserving protocols for sealed-bid auctions. *Computers & Security*, s. 101502.

Blass, E.-O., & Kerschbaum, F. (2018). Strain: A secure auction for blockchains. *European Symposium on Research in Computer Security* (s. 87-110). Cham: Springer International Publishing.

Braghin, C., Cimato, S., & Damiani, E. (2020). Designing smart-contract based auctions. Security with Intelligent Computing and Big-data Services: Proceedings of the Second International Conference on Security with Intelligent Computing and Big Data Services (SICBS-2018) (s.54-64). Springer : International Publishing.

Chen, Y.-H., Chen, S.-H., & Lin, I.-C. (2018). Blockchain based smart contract for bidding system. *IEEE International Conference on Applied System Invention (ICASI)* (s. 208-211). IEEE.

Chin, K., Emura, K., SATO, S., & OMOTE, K. (2024). A sealed-bid auction with fund binding: Preventing maximum bidding price leakage. *IEICE TRANSACTIONS on Information and Systems* , 615-624.

Desai, H., Kantarcioglu, M., & Kagal, L. (2019). A hybrid blockchain architecture for privacy-enabled and accountable auctions. *IEEE International Conference on Blockchain (Blockchain)* (s. 34-43). IEEE.

Dreier, J., Lafourcade, P., & Lakhnech, Y. (2013). Formal verification of e-auction protocols. *International Conference on Principles of Security and Trust,* (s. 247-266). Berlin, Heidelberg: Springer.

Galal, H., & Youssef, A. (2019). Verifiable sealed-bid auction on the ethereum blockchain. *Financial Cryptography and Data Security: FC 2018 International Workshops, BITCOIN, VOTING, and WTSC* (s. 265-278). Berlin Heidelberg: Springer .

Germouty, P., Larraia, E., & Zhang, W. (2023). Publicly Verifiable Auctions with Privacy. *International Conference on Financial Cryptography and Data Security* (s. 420-438). Switzerland: Springer Nature .

Goldwasser, S., & Micali, S. (2019). Probabilistic encryption & how to play mental poker keeping secret all partial information. Providing sound foundations for cryptography.

Gysel, M., Ford, B., & Zhang, H. (2023). *Sealed-Bid Blockchain Auctions using Flash Freezing Flash Boys.* Lausanne: École Polytechnique Fédérale de Lausanne (EPFL).

Keizer, N., Ascigil, O., Psaras, I., & Pavlou, G. (2021). Flock: Fast, lightweight, and scalable allocation for decentralized services on blockchain. *IEEE International Conference on Blockchain and Cryptocurrency (ICBC)* (s. 1-9). IEEE.

Li, H., & Xue, W. (2021, May 19). A blockchain-based sealed-bid e-auction scheme with smart contract and zero-knowledge proof. *Security and Communication Networks*, s. 1-10.

Liu, X., Liu, L., Yuan, Y., Long, Y.-H., Li, S.-X., & Wang, F.-Y. (2024). When Blockchain Meets Auction: A Comprehensive Survey. *IEEE Transactions on Computational Social Systems*, 1-13.

S. S., S., Khandelwal, N., Nathwani, M., Munot, P., & Patil, S. (2022). A Survey of E-bidding System using Blockchain. *4th International Conference on Smart Systems and Inventive Technology (ICSSIT)* (s. 250-255). IEEE.

Shi, Z., Laat, C., Grosso, P., & Zhao, Z. (2022). Integration of blockchain and auction models: A survey, some applications, and challenges. *IEEE Communications Surveys & Tutorials*, 497-537.

Trevathan, J. (2005). Security, anonymity and trust in electronic auctions. The ACM Magazine for Students.

Ye , Z., Chen, C.-L., Weng, W., & Sun, H. (2023). An anonymous and fair auction system based on blockchain. *The Journal of Supercomputing* , 13909-13951.

Zhang, M., Yang, M., & Shen, G. (2022). SSBAS-FA: A secure sealed-bid e-auction scheme with fair arbitration based on time-released blockchain. *Journal of systems architecture* , 102619.

# CHAPTER III

# Statistical Analysis on The Impact of Socioeconomic Factors on Student Education and Academic Achievement

**Sajedeh NOROZPOUR**[1]

## 1.Introduction

From the social, cultural, scientific, economic, and comprehensive national point of view, education is considered the most important aspect in developing countries. Education depends on the socio-economic factors of a country that directly affect the students. Many variables that affect the level of education and student achievement are also present in higher education. At private colleges, most students come from diverse socioeconomic backgrounds. Their academic success is affected by some of the issues that have limited their access to higher education. The purpose

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of this study was to examine how socioeconomic factors may affect the educational outcomes of students at private colleges. We used multiple universities and students to obtain data. We designed a hypothetical model that included several hypotheses to elicit student response. After the extensive literature review, we extracted the hypotheses and based on these hypotheses, this study defined the dependent and independent variables. In this study, 10 hypotheses and several variables were used to reveal the socioeconomic factors. These variables have a direct influence on education or learning under socioeconomic variations. To determine the results of the defined hypotheses, statistical techniques, Cronbach's alpha, and descriptive analysis were used in this study. In addition, we used the skewness and kurtosis values in our tests.

## Literature Review

Education is crucial for the growth of human capital and is associated with a person's happiness and chances for a higher quality of life. According to the concept of human capital, investment in education leads to the development of human capital, which is a critical component of economic growth. Education and training transform intellectual resources by transferring skills and useful information. Human capital is the pool of skills and useful knowledge that people possess. The best gift a country can give its citizens, especially its young people, is education. This is because a country's ability to develop depends to a large extent on the quality of its education system. Education is not only related to well-being and prospects for a better life, but is also crucial for the growth of human capital. According to the human capital hypothesis, investment in education leads to the development of human capital, which is a critical component of economic growth. Human capital is transformed through education and training, which also provide skills and useful information. Human capital consists of the skills and expertise that can be used.

In a study of factors influencing student academic achievement, it was shown that in addition to educational factors, social factors, and student factors, family characteristics, including socioeconomic status, are the strongest determinants of student academic achievement. Although parents' attitudes have a lesser effect on their students' academic performance than their education, family background also has an impact on students' academic performance. Academic performance is highly influenced by students, with better performance in all subjects having a cumulative effect (Aikens and Barbarin, 2008).

Parental poverty is a significant barrier to student growth and success in the classroom. Various socioeconomic factors in parents, families, and networks can be used to predict students' academic achievement. They examined how socioeconomic factors in southern Africa affect students' academic achievement. Similar findings were made in Africa, where higher-income parents send their students to institutes rather than public elementary school (Aikens and Barbarin, 2008).

The total number of students applying to and enrolling in universities has changed in recent years, becoming more diverse in terms of social, cultural, and economic capital, as well as age, nationality, prior education, and academic achievement. Although there is still a long way to go, the participation of learners from disadvantaged economic and social backgrounds in higher education is increasing. This is because in several countries socioeconomic developments have also contributed to improvements in higher education institutions. Some scholars have examined the relationship between socioeconomic status (SES) and educational attainment in hopes of better understanding such trends within higher education systems and have found a minimal to significant relationshipSociological studies have demonstrated a strong relationship between a family's socioeconomic status (SES) and the academic success of its students. One study showed that students from low socioeconomic backgrounds performed worse academically than students from high socioeconomic backgrounds, although the optimal method for measuring socioeconomic status is not clear. However, most research compares students from all socioeconomic strata and concludes that low socioeconomic position has a negative impact on various academic outcomes (Bao et al., 2006).

However, a crucial aspect is the variables that may influence academic performance within specific SES domains. The relationship between students' socioeconomic situation and their academic ability is one of the issues that educators argue about most. A common claim is that a student's socioeconomic status has a significant impact on his or her academic success. National audits have shown that academic achievement is generally low in many secondary schools in areas with a high proportion of students from socioeconomically disadvantaged backgrounds. Many educators believe that low socioeconomic status has a detrimental effect on student achievement. The basic needs of some students are not met, making it impossible for them to perform academically or physically. Students cannot be expected to perform well in school if they are not adequately fed or sanitised. Teachers believe that these environmental deficiencies negatively affect students' identities and diminish their self-esteem. This lack of self-confidence hinders a student's ability to succeed in the classroom. Some dispute this hypothesis, claiming that factors other than a student's socioeconomic status determine academic achievement. This study collects data on the educational achievement of students from different socioeconomic backgrounds and examines variations in traditional SES measures as well as a variety of other family, individual, and environmental factors (Beller and Hout, 2006).

According to the theory of educational productivity, school is the key factor that positively affects students' academic progress. Success is influenced by a variety of factors beyond curriculum, teaching methods and practices. Student psychological makeup, parental involvement, and community support all have a significant impact on student academic achievement. The various factors that affect academic achievement can be identified using Walberg's theory. According to the study, living situation is the most important component for the academic achievement of students from low socioeconomic position. It is more important than other elements that influence academic success, such as parental income and educational attainment. (Walberg, 1980).

Although the quality of engagement and income cannot be changed, universities can improve the living environment through instruction and collaboration with parents. According to one study, students' involvement in their educational process is significantly influenced by their family's socioeconomic level (Blau and Duncan, 1967) (Bornstein and Bradley, 2003).

It is important to focus on low-income students for several reasons. First, they are much less likely to attain a college degree. Although the expansion of higher education over the 20th century has increased opportunities for learners from all socioeconomic backgrounds, the focus of policy and research has increasingly shifted to student retention and graduation disparities-disparities that often begin in the first year. First-generation students from low-income families are about four times more likely to drop out after their first year than their peers from more affluent backgrounds. In addition, socioeconomically disadvantaged students often experience academic difficulties that result in lower grades and fewer credits earned. Second, the families of students with low socioeconomic status are often characterized as unable to support them on the path to success when family support is considered in previous research related to them. Most of the literature in this area focuses on the transition to higher education and highlights the difficulties faced by economically disadvantaged students and their families due to the lack of information and knowledge needed to successfully navigate higher education. Socioeconomically disadvantaged parents typically leave most of the decision-making to students, while economically advantaged parents are often actively involved in the college decision-making process. Socioeconomically disadvantaged parents tend to be less involved in their students' academic decisions and experiences than more affluent parents, and this pattern continues after college admission (Bornstein and Bradley, 2003) (Bowey and Judith, 1995).

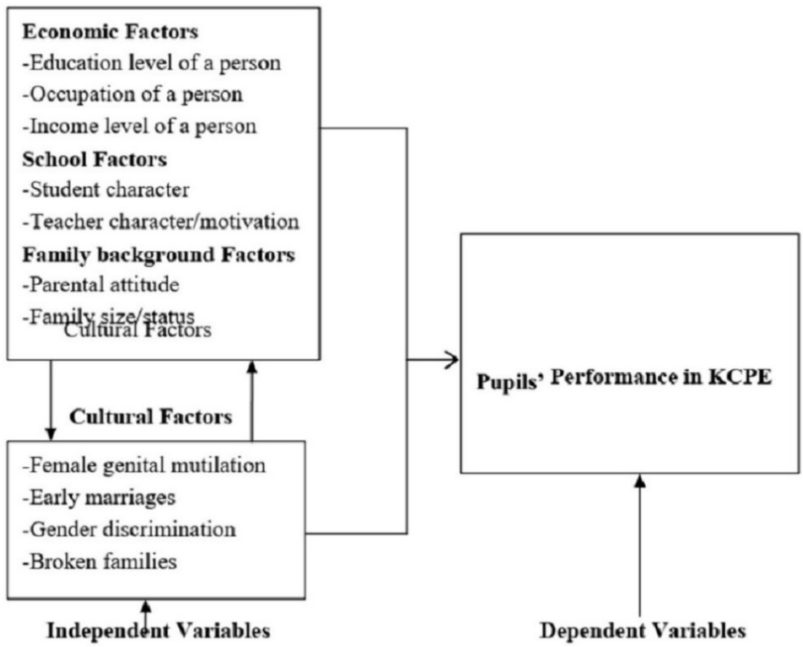
Strengths-based strategies emphasize the particular strengths, prospects, and persistence of disadvantaged groups rather than focusing on the deficits of disadvantaged families. Historically underrepresented racial and ethnic groups have been the focus of most previous studies on this topic. To refute the deficit frame that dominates much of the literature, one study says it is important to ground research in the lived experiences and expertise of people of color (Bradley and Corwyn, 2002) (Breen and Jonsson, 2005) (Chi, Wang and Cao, 2016). Research that carefully examined the function of family support contradicted the deficit approach and demonstrated the critical role parents play in helping minority students succeed in college. According to some studies, reading habits and socioeconomic status affect students' academic performance. By describing variables such as students' age, parental education, parental occupational prestige, family income, and household resources, SES is taken into account. It was found that there is a positive correlation between the three SES variables of income, occupation, and education (Chiu et al., 2015) (Ciping et al., 2015) (Liu, Luo and Liang, 2020).

According to various empirical studies, each of these elements represents a very specific aspect of SES. It has been argued that the relationship between family income and student education has become stronger over time and that "money has a causal influence on educational outcomes" One of the most important SES factors is parental education, as it affects students from early life through adolescence and beyond (Liu, Luo and Liang, 2020).

## Conceptual Framework

A conceptual framework serves as a model illustrating the relationships between various variables, particularly independent and dependent variables. Figure 1 depicts the conceptual framework used to explain the interconnections among the variables under study. The framework demonstrates that multiple factors impact student success, including financial aspects such as educational attainment, occupation, and monetary resources, as well as personal attributes like parental attitudes, teacher effectiveness, and family dynamics including size and prestige. Cultural factors such as female genital mutilation, early marriage, gender disparities, and familial disruptions may also influence academic achievement. The premise is that by analyzing these socioeconomic parameters, one can forecast the criterion variable, which in this case is student achievement. However, governmental regulations, guidance, and counseling may potentially diminish productivity levels. These mediating variables affect both the independent factors and the dependent variable within the framework.

*Figure 1: Scioeconomic Factors*




## Research Methodology

A research methodology is a set of guidelines for data collection and analysis that seeks to balance relevance with study purpose. It is the conceptual framework within which research is conducted. To determine how socioeconomic factors, affect the academic achievement of students in educational settings, the study used a descriptive approach. Descriptive research is a data collection technique in which a group of respondents are interviewed or surveyed using a questionnaire. Questionnaires were used in the study to collect data. However, questionnaires are a carefully designed tool to collect data that meet the requirements of the research questions. Since the researcher was only to describe the socioeconomic aspects that affect students' educational success without changing the variables, the informative survey approach was appropriate for this study.

In the context of the study, the target group is the entire population or the entire study area. The target group consisted of 3000 students from 5 private educational institutions, 110 classroom teachers. In a descriptive study, a sample of 10% to 20% of the population is appropriate, but the larger sample, the more meaningful. It argues that a sample must be large enough to represent an acceptable proportion of the population about which the researcher wishes to generalize, and small enough to determine the accessibility of subjects and the cost in terms of time and money.

Internal reliability and the degree of relatedness within a set of items are measured using Cronbach's alpha, which serves as an indicator of level stability. In scientific contexts, Cronbach's alpha is referred to as the reliability coefficient. We conducted Cronbach's alpha tests as outlined in Table 1. These tests elucidate the magnitude and relationship of each hypothesis with the predictor variable, shedding light on how the stated hypotheses impact the results of the dependent variable. In this instance, the Cronbach's alpha result exceeds 0.7. Additionally, the average variance extracted (AVE) for each variable, which is consistently and statistically reliable, surpasses 0.50. Consequently, higher values for each variable signify a strong correlation between the hypothesis and the predictor variable.

*Table 1: Cronbach's Alpha, Composite Reliability and Average value*

| Variable | Cronbach's Alpha | Composite Reliability | (AVE) |
|---|---|---|---|
| Economic factor | .514 | .715 | .75 |
| School factor | .652 | .552 | .74 |
| Family background factor | .704 | .725 | .82 |
| Cultural background | .748 | .741 | .79 |

**Descriptive Analysis and Result**

Descriptive statistics serve to delineate the primary characteristics of a study's findings, offering a rapid overview of the measured variables and the sample. Descriptive analysis is employed to showcase the values corresponding to the hypotheses posited in our survey research. The participants' responses are elucidated within the hypothesis results. The Table 2 elucidates the descriptive analysis test.

In total, our study encompasses 10 hypotheses examining the direct relationships between the four groups of economic, school, family background, and cultural background factors on the education and career outcomes of students. Additionally, we have formulated 6 comparison hypotheses to assess which variable exerts a greater influence on students' career trajectories.

H1:

- Null Hypothesis (H0): There is no significant relationship between economic factors and academic performance.
- Alternative Hypothesis (HA): Economic factors have a significant impact on academic performance.

H2:

- Null Hypothesis (H0): There is no significant relationship between school factors and academic performance.
- Alternative Hypothesis (HA): School factors have a significant impact on academic performance.

H3:

- Null Hypothesis (H0): There is no significant relationship between family background factors and academic performance.
- Alternative Hypothesis (HA): Family background factors have a significant impact on academic performance.

H4:

- Null Hypothesis (H0): There is no significant relationship between Cultural background factors and academic performance.
- Alternative Hypothesis (HA): Cultural background factors have a significant impact on academic performance.

H5:

- Null Hypothesis (H0): There is no significant difference in the impact of family background factors and cultural background factors on students' career aspirations.
- Alternative Hypothesis (HA): Family background factors have a greater impact on students' career aspirations compared to cultural background factors.

H6:

- Null Hypothesis (H0): There is no significant difference in the impact of family background factors and economic factors on students' career aspirations.
- Alternative Hypothesis (HA): Family background factors have a greater impact on students' career aspirations compared to economic factors.

H7:

- Null Hypothesis (H0): There is no significant difference in the impact of family background factors and school factors on students' career aspirations.
- Alternative Hypothesis (HA): Family background factors have a greater impact on students' career aspirations compared to school factors.

H8:

- Null Hypothesis (H0): There is no significant difference in the impact of economic factors and cultural background factors on students' career aspirations.
- Alternative Hypothesis (HA): Economic factors have a greater impact on students' career aspirations compared to cultural background factors.

H9:

- Null Hypothesis (H0): There is no significant difference in the impact of school factors and cultural background factors on students' career aspirations.
- Alternative Hypothesis (HA): School factors have a greater impact on students' career aspirations compared to cultural background factors.

H10:

- Null Hypothesis (H0): There is no significant difference in the impact of school factors and economic factors on students' career aspirations.
- Alternative Hypothesis (HA): economic factors have a greater impact on students' career aspirations compared to school factors.

*Table 2: Descriptive analysis of hypotheses*

| Hypotheses | Mean | SE | SD | SK | KU |
|---|---|---|---|---|---|
| H 1 | 0.71 | 0.34 | 0.25 | -1.7 | 1.2 |
| H 2 | 0.67 | 0.34 | 0.36 | -1.0 | -0.5 |
| H 3 | 0.62 | 0.34 | 0.29 | -0.7 | -1.1 |
| H 4 | 0.45 | 0.34 | 0.53 | -0.2 | -.15 |
| H 5 | 0.71 | 0.34 | 0.47 | -.15 | -1.7 |
| H 6 | 0.67 | 0.34 | 0.57 | -.32 | -1.6 |
| H 7 | 0.72 | 0.34 | 0.49 | -.17 | -1.4 |
| H 8 | 0.72 | 0.34 | 0.41 | -0.12 | -1.1 |
| H 9 | 0.52 | 0.34 | 0.42 | -02.5 | 2.4 |
| H 10 | 0.72 | 0.34 | 0.50 | -2.3 | 1.8 |

Hypotheses 1, 2, 3 5, 6, 7, 8, and 10 demonstrated relatively high mean values, suggesting a positive relationship between socioeconomic factors and academic performance. However, the standard deviations varied across these hypotheses, indicating some degree of variability within the sample. Skewness and kurtosis values were generally within acceptable ranges, suggesting normally distributed data for these hypotheses.

Conversely, Hypotheses 4 and 9 showed lower mean values, indicating weaker associations between the variables under investigation. The standard deviations were also notable in these cases, indicating greater variability within the sample. Skewness and kurtosis values for these hypotheses suggested non-normal distributions, potentially indicating underlying complexities in the relationship between socioeconomic factors and academic achievement.

Overall, while some hypotheses supported the notion of socioeconomic factors influencing academic performance, others revealed nuances and complexities in this relationship. These findings underscore the need for comprehensive approaches to address the multifaceted nature of student success in educational settings.

Moving forward, educational policymakers and practitioners should consider the differential impact of socioeconomic factors on academic achievement and tailor interventions accordingly. Additionally, further research is warranted to explore the underlying mechanisms driving these associations and to inform evidence-based strategies for promoting educational equity and excellence.

## Conclusion

In conclusion, this study sheds light on the significant influence of family background and socioeconomic factors on students' educational outcomes. It underscores the challenges students face due to parental income and financial constraints, which hinder their ability to pursue education effectively. Moreover, inadequate educational facilities and funding exacerbate the situation, leading to higher dropout rates. The government's role in addressing these issues by implementing policies that alleviate financial burdens and promote access to education is crucial.

Furthermore, the correlation between parental wealth and student academic performance highlights the need for targeted interventions to mitigate disparities. Overall, understanding the multifaceted impact of socioeconomic factors on students' educational trajectories is essential for devising effective strategies to support their academic success.

## References


Aikens, N. L., & Barbarin, O. (2008). Socioeconomic differences in reading trajectories: The contribution of family, neighborhood, and school contexts. Journal of educational psychology, 100(2), 235.

Bao, S., Chang, G. H., Sachs, J. D., & Woo, W. T. (2002). Geographic factors and China's regional development under market reforms, 1978–1998. China Economic Review, 13(1), 89-111.

Beller, E., & Hout, M. (2006). Welfare states and social mobility: How educational and social policy may affect cross-national differences in the association between occupational origins and destinations. Research in social stratification and mobility, 24(4), 353-365.

Walberg, H. J. (1980). A psychological theory of educational productivity. National Inst. of Education (DHEW), Washington, DC.

Blau P. M, Duncan O. D. (1967). The American occupational structure. New York: Wiley Press.

Bornstein, Bradley R. H. (2003). Socioeconomic status, parenting, and child development. Mahwah, NJ: Lawrence Erlbaum.

Bowey, J. A. (1995). Socioeconomic status differences in preschool phonological sensitivity and first-grade reading achievement. Journal of educational Psychology, 87(3), 476.

Letourneau, N. L., Duffett-Leger, L., Levac, L., Watson, B., & Young-Morris, C. (2013). Socioeconomic status and child development: A meta-analysis. Journal of Emotional and Behavioral Disorders, 21(3), 211-224.

Breen, R., & Jonsson, J. O. (2005). Inequality of opportunity in comparative perspective: Recent research on educational attainment and social mobility. Annu. Rev. Sociol., 31, 223-243.

迟少辉, 王祖浩, & 曹彬彬. (2016). 不同科学能力学生家庭社会经济地位的差异研究[socio-economic status differences among the students with the different levels of science competencies]. 全球教育展望, (2), 113-128.

Chiu, M. M., & Xihua, Z. (2008). Family and motivation effects on mathematics achievement: Analyses of students in 41 countries. Learning and instruction, 18(4), 321-336.

Ciping, D., Silinskas, G., Wei, W., & Georgiou, G. K. (2015). Cross-lagged relationships between home learning environment and academic achievement in Chinese. Early Childhood Research Quarterly, 33, 12-20.

Liu, J., Peng, P., & Luo, L. (2020). The relation between family socioeconomic status and academic achievement in China: A meta-analysis. Educational Psychology Review, 32, 49-76.

# CHAPTER IV

# On Similarity and Poloid

**Hasan KELEŞ**[1]

## 1.Introduction

This study is about similarity and poloid. Some properties of these two structures, which have common features, are investigated. A monoid is the smallest structure of a group structure. In this smallest structure, all elements have inverse. These inverse elements are only one. The poloid is a new concept. A poloid is a small expansion of a monoid. All elements have inverse in monoid. At the same time, in this poloid structure, each element is a product of three elements, at least one of which is common and two of which are different. That is, A=BP=PC, where B≠C. The concept of similarity is used in matrices. An example of a poloid is the structure of regular

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matrices. The concept of similarity is used in matrices. Many application areas of similar matrices are possible. An example of a poloid is the structure of regular matrices. This situation is the intersection point for the two topics. This idea is the main idea of the study. The structure is analysed again for similar mantrices in the target. Known properties, lemmas, theorems are explained. This information and the definition of poloid, lemmas and theorems belonging to poloid are provided new contributions to the study. The definition of similarity of poloid structure is given. Poloid structure related to similarity is analysed. New lemmas about similarity are given by using the transitivity property of the poloid. Different approaches to the application areas of similarity are gained. This investigation is theoretical. There are some approaches for application areas.

Let $\mathbb{F}$ be a field and $M_n(\mathbb{F})=\left\{\left[a_{ij}\right]_n \middle| a_{ij}\in\mathbb{F}, n\in\mathbb{Z}^+\right\}$.
Suppose $A,B,C\in M_n(\mathbb{F})$. Then

i. $A$ is similar to $A$. (Reflexive)
ii. If $A$ is similar to $B$, then $B$ is similar to $A$. (Symmetric)
iii. If $A$ is similar to $B$ and $B$ is similar to $C$, then $A$ is similar to $C$. (Transitive)

Similarity is an Equivalence Relation. The fact that similarity is an equivalence relation is not emphasized here. Briefly, in this study, the necessary and sufficient conditions of similarity are investigated. Obtained property, lemma and theorems are given. An infinite similar matrix is written to any matrix**.** The situation same is true for equivalent matrices in [1].

Is there some a matrix $A,B\in\mathbb{M}_n(\mathbb{F})$ for any $F\in\mathbb{M}_n(\mathbb{F})$ such that, $AB=F$? Also, are these $A,B$ matrices unique? This question is a topic worth researching for many years.

**Lemma 1.1.** If $B|F$, then there exists $A\in\mathbb{M}_n(\mathbb{F})$ such that,

$$BA=F \text{ or } F=BA.$$

Except for special cases in matrix multiplication, $AB\neq BA$.

**Definition 1.2.** Let $\mathbb{F}$ be a field and $\mathbb{M}_n(\mathbb{F})=\left\{[a_{ij}]_n \,\middle|\, a_{ij}\in\mathbb{F}, n\in\mathbb{Z}^+\right\}$. The rational matrix is defined by

$$-:\mathbb{M}_n(\mathbb{F})\times\mathbb{M}_n(\mathbb{F})\to\mathbb{M}_n(\mathbb{F})$$

$$(B,F)\to\frac{F}{B}:=\left[\frac{\left({}_B^F i_j\right)_{ji}}{|B|}\right]_{n\times n}$$, where $\left[\left({}_B^F i_j\right)_{ji}\right]_n$ is the column co-divisor matrix on the $B$ matrix of the matrix $F$.

The rational matrix is denoted by $\mathbb{Q}(\mathbb{M}_n(\mathbb{F}))$.

The matrix $\left[\left({}_B^F i_j\right)_{ji}\right]_n$ is the determinant of the matrix obtained by writing the $i^{th}$ column of the $F$ matrix into the $j^{th}$ column of the $B$ matrix in [3].

**Definition 1.3.** The determinant of the new matrix obtained by writing on the $j^{th}$ row of the matrix $B$ the $i^{th}$ row of the matrix $A$ is called the *co-divisor by row* of the matrix $A$ by the row on the matrix

$B$. It is denoted by $\underset{ij}{AB}$. Their number is $n^2$. The matrix co-divisor by row is $\left[\left(\underset{ij}{AB}\right)_{ij}\right]$, where $A,B\in\mathbb{M}_n(\mathbb{F})$ [3].

**Theorem 1.4. ([3]).** Let $\frac{A}{B}\in\mathbb{Q}(\mathbb{M}_n(\mathbb{F}))$ Then, for least the regular matrix $A_1\in\mathbb{M}_n(\mathbb{F})$ exists such that $A=A_1A_2$ and $B=A_1B_2$ it satisfies equation,

$$\frac{A}{B}=\frac{A_1A_2}{A_1B_2}=\frac{A_2}{B_2}, \text{ where } A_2,B_2\in\mathbb{M}_n(\mathbb{F}).$$

Avital made important contributions to mathematics in his time [2]. Two authors J. C. Evard, and J. M. Gracia are conducted research on the applications of the study of similarity in the 1990s [2]. Many mathematical scientists are made important contributions to mathematical science on the subject in [3, 4, 5, 6, 7, 9, 10, 11]. The definition of poloid is derived from my studies in 2022.

**Definition 1.5.** A group is a set $\mathbb{P}$ equipped with a binary operation $*:\mathbb{P}\times\mathbb{P}\to\mathbb{P}$ that associates an element $a*b\in\mathbb{P}$ to every pair of elements $a,b\in\mathbb{P}$, and having the following properties: $*$ is associative, has an identity element $e\in\mathbb{P}$, and every element in $\mathbb{P}$ is invertible More explicitly, this means that the following equations hold for all $a,b,c,d\in\mathbb{P}$:

$P1$. $a*(b*c)=(a*b)*c$, (associativity).

$P2$. $a*e=e*a$, (identity).

$P3$. For every $a\in\mathbb{P}$, there is some $a^{-1}\in\mathbb{P}$ such that $a*a^{-1}=a^{-1}*a=e$ (inverse).

A set $\mathbb{P}$ together with an operation $*:\mathbb{P}\times\mathbb{P}\to\mathbb{P}$ and an element satisfying only conditions (P1), (P2) and (P3) is called a group.

This structure is called poloid, when the following property (P4) is added in addition to these properties in [8].

$P4$. For any $a\in\mathbb{P}\setminus\{e\}$, there exists $b,c,d\in\mathbb{P}\setminus\{e\}$ such that

$\underbrace{b*\underbrace{c=c}_{=}*d}_{\neq}=a$ with $b\neq d$ (escort).

If it satisfies the following property then a $(\mathbb{P},*)$ monoid is called a poloid,

For any $a\in\mathbb{P}$ there some exists $b,c,p\in\mathbb{P}$ such that $a=b*p=p*c$, where $b\neq c$ in [9].

**Lemma 1.6.** Let $(\mathbb{P},*)$ a poloid. If $a\in\mathbb{P}$, then there exists $b,c,d,e,p,q\in\mathbb{P}$ such that $a=b*p=p*c=d*q=q*e$, where $b\neq c, d\neq e$.

**Proof.** The proof is clear.

The definition of similarity in poloid is given.

**Definition 1.7.** For any two matrices $a,b\in\mathbb{P}$ are said to be similar or conjugated if they are the same linear transformation in possibly different bases,

$$a=p^{-1}*b*p$$

If $a$ similar to $b$ then it is denote as $a\sim b$.

The following lemma is necessary for poloid.

**Lemma 1.8.** Let $a,b,c,d \in \mathbb{P}\setminus\{e\}$.

i. If $a|b$ then $a*a|b$, where $b_i \in \mathbb{P}\setminus\{e\}$.
ii. If $b|a, c|a$, then $c*c_i = b*b_i$, where $b_i, c_i \in \mathbb{P}\setminus\{e\}$.
iii. If $a = c*c_i = b*b_i$ for $c \neq b$ there are $b_i, c_i \in \mathbb{P}\setminus\{e\}$ then $c_i \neq b_i$.
iv. If $b|a$ then $b^{-1} = b_i * b * b_j$, where $b_i, b_j \in \mathbb{P}\setminus\{e\}$.

Proof. Let $(\mathbb{P},*)$ be a poloid and for all $a,b,c,d \in \mathbb{P}\setminus\{e\}$.

i. If $a|b$ then

$$b = a*a_i \text{ and } a|b_i$$

$$b{=}a*a_i = a*a*b_i, \text{ where } b_i \in \mathbb{P}\setminus\{e\}.$$

ii. If $b|a, c|a$ and by (P2) then

$$a = b*b_i, .\, a = c*c_i,$$

$$c*c_i = b*b_i, \text{ where } b_i, c_i \in \mathbb{P}\setminus\{e\}.$$

iii. If $a = c*c_i = b*b_i$ for $c \neq b$ there exist $b_i, c_i \in \mathbb{P}\setminus\{e\}$

$c_i \neq b_i$ by lemma (ii).

iv. If $b|a$ then $b|a^{-1}$.

And by (P3)

$$a = b * b_i ,\ a^{-1} = b * c_i ,$$

$$a * a^{-1} = b * \underbrace{b_i * b * c_i}_{b^{-1}} = e$$

$$b^{-1} = b_i * b * c_i .$$

The set of all regular matrices of order $n$ over a field $\mathbb{F}$ is denoted by $\mathbb{M}_n(\mathbb{F})$. The transpose of $A \in \mathbb{M}_n(\mathbb{F})$ is denoted by $A^T$.

**Definition 1.9.** Two matrices $A, B \in \mathbb{M}_n(\mathbb{F})$ are said to be similar or conjugated if they are the same linear transformation in possibly different bases,

$$A = P^{-1}BP$$

where $P$ is an invertible change of basis matrix. If $A$ similar to $B$ then it is denote as $A \sim B$ in [5, 7, 9, 10, 11].

**Similarity and Poloid**

In this section, we are called about the intersection of similarity and poloid. $a = b * p = p * c$ and $A = P^{-1}BP$ are received.

**Lemma 2.1.** Let $(\mathbb{P}, *)$ a poloid. If $a \sim b$ and $a \sim c$, then $b \sim c$, where $a, b, c \in \mathbb{P}$.

**Proof.** If $a \sim b$ then $a = p^{-1}bp$, where regular $p \in \mathbb{P}$.

If $a \sim c$ then $a = q^{-1}cq$, where regular $q \in \mathbb{P}$.

and

$$a = p^{-1}bp = q^{-1}cq$$

$$b = pq^{-1}cqp^{-1} = \left(qp^{-1}\right)^{-1} c\left(qp^{-1}\right)$$

$$b \sim c\,.$$

Let us start with the following lemma.

**Lemma 2.2.** If $A \in \mathbb{M}_n\left(\mathbb{F}\right)$ then there some exists $B_1, C_1 \in \mathbb{M}_n\left(\mathbb{F}\right)$ such that $A \sim B_1, A \sim C_1$.

**Proof**. If $A \in \mathbb{M}_n\left(\mathbb{F}\right)$ then $\left(\mathbb{M}_n\left(\mathbb{F}\right),.\right)$ is poloid and for any $A \in \mathbb{M}_n\left(\mathbb{F}\right)$ there some exists $B, C, P \in \mathbb{M}_n\left(\mathbb{F}\right)$ such that $A = BP = PC$, where $B \neq C$ .

$$B = P^{-1}B_1, C = C_1P^{-1},$$

$$A = BP = P^{-1}B_1P \Rightarrow A \sim B_1\text{, where } B_1 \in \mathbb{M}_n\left(\mathbb{F}\right).$$

$$A = PC = PC_1P^{-1} \Rightarrow A \sim C_1\text{, where } C_1 \in \mathbb{M}_n\left(\mathbb{F}\right).$$

**Corollary 2.3.** If $A \sim B_1, A \sim C_1$ then, $B_1 \sim C_1$.

**Proof.** If $A \sim B_1, A \sim C_1$ then, $B_1 \sim C_1$.

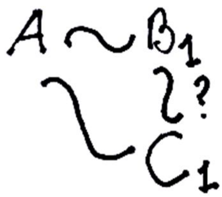


Figure 1. Triple similarity

$$A = P^{-1}B_1P, A = PC_1P^{-1}$$

$$P^{-1}B_1P = PC_1P^{-1} \Rightarrow B_1 = P^2C_1\left(P^2\right)^{-1}$$

$$B_1 \sim C_1 .$$

**Lemma 2.4.** If $A \in \mathbb{M}_n\left(\mathbb{F}\right)$ then There are more than two similar matrices of matrix $A$ .

**Proof**. If $A \in \mathbb{M}_n\left(\mathbb{F}\right)$ then $\left(\mathbb{M}_n\left(\mathbb{F}\right),.\right)$ is poloid and for any $A \in \mathbb{M}_n\left(\mathbb{F}\right)$ there some exists $B, C, D, E,\ Q, P \in \mathbb{M}_n\left(\mathbb{F}\right)$ such that

$$A = BP = PC\ ,\ A = DQ = QE\ \text{, where } B \neq C, D \neq E\ .$$

$$A \sim B_1, A \sim C_1, A \sim D_1, A \sim E_1 .$$

**Discussions**

Poloid is refined the method of obtaining similarity. Results for singular structures are not investigated. Obtaining the strong similarity condition is an open problem.

## Conclusıons

Similarity structure is transferred to Poloid structure in matrices. The following results are obtained.

i. The definition of similarity on poloid is given.

ii. Similar elements are varied according to the different element.

iii. The number of similar elements is infinite.

iv. Two similar elements similar to the same element are similar to each other

## References


Avital, S., (1995). History of Mathematics Can Help Improve Instruction and Learning, Mathematical Association of America, 3-12.

Evard, J. C. and Gracia, J. M., (1990). On Similarities of Class $C^P$ and Applications to Matrix Differential Equations, Linear Algebra and its Applications,(137–138), 363-386.

Keleş, H., (2022). On Some Results Row Co-Division in Regular Square Matrices, Current Debates on Natural and Engineering Sciences 3, Bilgin Culture and Art Publications, Certificate No: 20193, 65-69.

Starčič, T., (2022). Isotropy groups of the action of orthogonal similarity on symmetric matrices, Linear and Multilinear Algebra, Taylor & Francis Group.

https://doi.org/10.1080/03081087.2022.2044746.

T. Jiang, Cheng, X. and Ling, S., (2014). An Algebraic Relation between Consimilarity and Similarity of Quaternion Matrices and Applications, Journal of Applied Mathematics, Volume, Article ID 795203, 5 pages, 2014. http://dx.doi.org/10.1155/2014/795203.

Paul, A. F., (1977). On Strict System Equivalence and Similarity, International Journal of Control, 25(1), 5-10.

V. M. Prokip, (2018). On the Similarity of Matrices AB and BA Over a field, Carpathian Math. Publ. 10(2), 352–359, doi:10.15330/cmp.10.2.352-359.

Nicholson, W. K., (2021). Linear Algebra with Applications, Creative Commons License, University of Calgary.

Keleş, H., (2022). Poloids and Matrices, The Aligarh Bulletin of Mathematics,(4) **1**, 41-52.

Maxim, N., (2006). Notes on Linear Algebra and Matrix Analysis, Version 0.1.1, 4(3).

Keleş, H. (2015). Lineer Cebire Giriş-I-, Bordo Puplication, Trabzon, Turkiye.

Duane, A. P., (1976). Solvability of the Matric Equation AX = B, Linear Algebra and Its Applzcatzons, 13, 177-164.

# CHAPTER V

# Fundamentals of Ear Biometric Systems and Their Forensic Significance

**Nurdan SEZGİN[1]**
**Serel ÖZMEN-AKYOL[2]**

## Introduction

Ears are an important facial marker that can help with identification. Side images of the face, particularly partial images captured on CCTV footage where the suspect's face is hidden or covered up, can be compared to ear images of known criminals to help in criminal investigations (Rani et al., 2022). Ear biometrics can precisely identify an individual using comparative analysis of the human ear and its morphology (Ogoun, 2022). Ear recognition relies

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on holistic and model-based approaches, with current work using deep learning. While ears can be disguised by hair, there are obvious advantages to adopting ears for biometric systems because they are impervious to expression (Meng & Nixon, 2021).

Biometric technology aims to identify individuals by imitating the human brain's pattern recognition process. Biometric technology uses physiological or behavioral variables to identify individuals (Dargan & Kumar, 2020). These abilities must meet specific criteria, such as universality and performance (Benzaoui et al., 2023).

Biometrics, which is concerned with identifying individuals based on their behavioral and physiological characteristics, is a reliable identification system that distinguishes one person from another by using physiological characteristics such as facial image, ear, retina, iris, fingerprint, hand geometry, and behavioral characteristics such as signature, voice, gait, and keystroke (Iyer et al., 2020; Anwar, Ghany & Elmahdy, 2015; Omara et al., 2017; Alkababji & Mohammed, 2021).

Biometric data, which is important in personal identification verification, forensic science, and security applications, is permanent, unforgettable, and nearly hard to replicate or recreate (Minaee et al., 2023; Dodge, Mounsef & Karam, 2018). Biometric methods have several major advantages, including uniqueness, convenience of data collecting, and privacy protection (Dodge, Mounsef & Karam, 2018).

In general, all biometric systems include 2 steps in practice (Iyer et al., 2020):

*The first step* is the registration phase, where new user data is added to the database for the first time;

*The second step* is the recognition phase, which is converted into a template using the same database and includes a positive or negative match. The recognition phase includes the processes of authentication (verifying a person's identity, 1 to 1) and identification (finding a person's identity, 1 to N) (Iyer et al., 2020).

User authentication is used by researchers as well as industries. Verification and identification procedures have become increasingly complex (Liang, Samtani & Yu, 2020). Authentication systems are divided into three group (Liang, Samtani & Yu, 2020; Ometov et al., 2019; Jain, Ross & Pankanti, 2006):

1. Knowledge-Based

2. Physiological biometric based

3. Behavioral biometric-based solutions.

To validate a person's identity with knowledge-based authentication, the user must provide a password, personal identification number (PIN), and graphic PIN. Physiological biometric authentication recognizes identities by using biological traits such as the facial image, iris, and fingerprint, when combined with machine learning algorithms. User authentication is also based on behavioral biometrics, such as gait, keystroke, and touchscreen Dynamics (Liang, Samtani & Yu, 2020).

Nowadays, there's plenty of interest in building identification systems that can be used to recognize humans on their own. Biometric applications in a variety of fields, from forensic science to national security, are quickly becoming a vital aspect of modern culture. Among these systems, fingerprints and face images are the biometrics with the biggest market share (Abaza et al., 2013).

Forensic comparative studies investigate an individual's unique and specific characteristics in order to determine their identification. The physical features of the human external ear vary and might be known different characteristics (Rani et al., 2022).

Various techniques have been created to identify biometric features for use in a variety of applications, including forensic investigations and security (Booysens & Viriri, 2022). Researchers from various fields are improving current technology and developing new identification methods that use the ear as supporting data. Ear examinations based on morphological and morphometric deviations at certain landmarks can be used as supporting evidence for personal identification in forensic cases. There are no commercially marketed ear recognition systems. Even as researchers work to improve the technology, there is huge opportunity in the future to add ear images with facial images in a multi-biometric system (Chhikara & Saini, 2023).

Ear biometrics can be applied to automated human identification and verification systems. Forensic science is one of the most common applications of this technology in criminal investigation and recognition (Sharkas, 2022).

In this study, it was investigated in which steps ear biometric data, which is a popular topic, is processed in artificial intelligence applications, and the most important of these steps, preprocessing and feature extraction, were emphasized.

## Forensic Significance

The ear was introduced as a method of human identification in the late nineteenth century, when Alphonse Bertillon included it as one of eleven anthropometric measurements in the manual technique, he used to identify people (Dhanda, Badhan & Garg, 2011). Ear biometrics is an important subject in forensics because, in a criminal investigation, ear prints and measurements can be used to verify identity if an accurate fingerprint is unavailable for examination (Verma et al., 2016; Fakorede et al., 2021).

Ear biometrics, which use comparative analysis of the human ear and its morphology, are useful in the identification of individuals. It is stated that the size of the pinna varies amongst different ethnic groups (Kumar & Singla, 2013). The human ear is morphologically diverse. However, data on differences between ethnic groups are insufficient, which is critical for identifying individuals in forensic sciences (Verma, Bhawana & Kumar, 2014; Chattopadhyay & Bhatia, 2009). A number of investigations in forensic anthropology have shown that the auricle is a useful biometric for identification, depending on anatomical variance in age, regional origin, and sex (Murgod et al., 2013).

The forensic science community is currently interested in the human ear as a key biometric for human identification and classification (Verma et al., 2016; Rubio et al., 2017). The human ear is distinctive to each individual, and ear prints, like fingerprints, can differentiate even identical twins (Chang, Bowyer & Barnabas, 2003; Rahman et al., 2007; Daramola & Oluwaninyo 2011). Some studies imply that the outer ear can be utilized to identify both living and deceased people (Swift & Rutty 2003; Abbas & Rutty 2005; Krishan et al., 2019).

The ear is more useful for identification since it is not as affected by situations like aging, changes in facial expression, and the use of glasses than the face (Victor, Bowyer & Sarkar, 2002; Hurley, Nixon & Carter, 2005).

**General structure of ear**

The ear develops in an orderly way throughout the first four months following birth. The ear then stretches throughout development between the ages of four months and eight years. Following this, the ear size and shape remain stable until the age of seventy, when they expand in size once more (Booysens & Viriri, 2022). According to certain research, significant changes in ear morphology occur before the age of eight and after the age of seventy (Sforza et al., 2009; Yoga et al., 2017). During this time, the ear grows with age, but the form of its parts is usually stable (Benzaoui et al., 2023).

The human ear includes of the helix, concha, tragus, lobe, and antitragus (Figure 1). The most essential part of the ear's exterior anatomy is the helix, which is divided into three parts: superior, anterior, and posterior. Individuals have unique helix structures. As a result, it is considered as an important component for identification in criminal investigations (Khobragade, Mor & Chhabra, 2015).

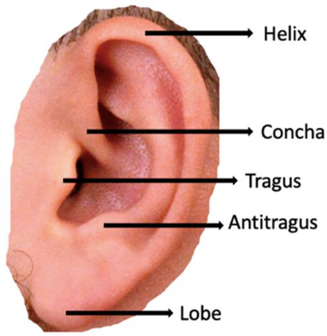



***Figure 1:*** *External ear*

The ear's structure is defined as both distinctive and permanent. As a result, its appearance remains consistent throughout a person's life. Furthermore, obtaining an image of someone's ear does not require their consent. Due of these features, curiosity about ear recognition systems has increased in recent years (Saranya et al., 2016).

The application areas of ear biometry used for security purposes are as follows (Khobragade, Mor & Chhabra, 2015):

- For security purposes in banks and ATMs
- For any active identification
- If ear images are obtained from security cameras for forensic purposes (Khobragade, Mor & Chhabra, 2015).

**Ear biometrics system**

The ear biometric system is a standart pattern recognition system in which the image being processed is reduced to a set of features that are then compared with feature sets from other images to identify them. This system has two operational modes:

verification mode, in which the user claims an identity, and identification mode, in which the user does not claim an identity (Abaza et al., 2013).

**Verification mode:** To verify the claim, the image being used is compared to the claimed identification through the use of relevant feature sets.

**Identification mode:** The submitted ear image is compared to a set of categorized ear images in a database to determine the best match and thus the identification (Abaza et al., 2013).

The stages of a classical ear recognition system include the following steps (Abaza et al., 2013):

1. Segmentation (ear detection): The first and most critical stage is establishing the location of the ear in the image. A rectangular boundary is commonly employed to determine the dimension of the ear. Ear detection is a vital component, as failures at this stage might reduce the biometric system's efficacy.

2. Normalization and enhancement: At this point, the scanned ear is subjected to an enhancing technique, which improves image fidelity.

3. Feature extraction: This is the process of converting the segmented ear into a mathematical model (such as a feature vector) that describes its distinctive properties.

4. Matching: The features extracted in the previous stage must be compared to the features recorded in the database. A match score is calculated by comparing the feature sets of two ear images. The match score indicates the similarity of the two ear images.

5. Decision: The final choice is made by considering the scores generated in the matching module. The verification mode of operation gives the values "yes" or "no". "Yes" indicates a correct match, whereas "no" includes exclusion (Abaza et al., 2013).

This system generally consists of 4 steps (Khobragade, Mor & Chhabra, 2015):

1. Capture and preprocessing

2. Feature extraction

3. Storage

4. Comparison of ear shapes (Khobragade, Mor & Chhabra, 2015).

The most important of these steps are preprocessing and feature extraction (Khobragade, Mor & Chhabra, 2015). In a general ear biometric system, the test image is first preprocessed, then feature extraction is performed and the image is recorded in the database (Figure 2).

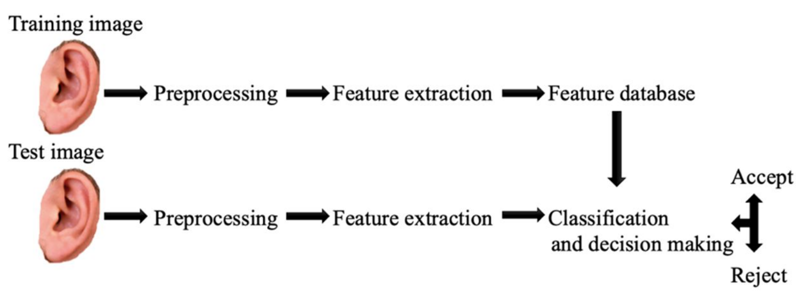



***Figure 2:*** *General ear biometric system*

Many countries are integrating biometric technologies into traditional identification verification systems, which are becoming a more trustworthy alternative. One of the most suitable candidates for use in biometrics is the ear. It does not change with emotions or cosmetic changes. Because it may be easily acquired from a distance, ear recognition is particularly useful for smart surveillance tasks as well as forensic image analysis (Saranya et al., 2016).

**Preprocessing**

In order to extract the features, the data must first be processed (Choras, 2007). In the preprocessing stage, which is one of the most important stages in ear biometric systems, the presence of the ear in the obtained image is detected and then the location of the ear in that image is determined. A weak ear detection stage can affect the subsequent stages and cause the system to produce incorrect results. Therefore, for an ear recognition system that is robust against exposure and occlusion, accurate and fast detection of the ear is very important (Alkababji & Mohammed, 2021).

In the preprocessing stage, 2 main categories are used when performing ear detection (Zhang & Mu, 2017):

1. Template matching: It involves detecting and locating the ear using R-CNN or other templates.
2. Shape-based: It involves identifying edges using the Hough transform (HT) and registering the ear based on the ear's elliptical shape (Zhang & Mu, 2017).

To process a digital image, the database is first integrated into the model. Then, the images are resized, converted to grayscale, smoothing and edge detection are performed (Nikose & Meena, 2020). The majority of visual data is incomplete, meaningless, and noisy. The main objective of image preprocessing is to remove excessively noisy and anomalous data that does not provide meaningful information. Data cleaning is the first step in data preprocessing. Noisy data is the most significant challenge to autonomous modeling. Without clean data, the models used will produce false results, substantially damaging the decision-making process. Data cleaning not only eliminates unnecessary data, but also corrects incorrect data in the dataset and reduces duplication (Ebanesar, Bibin & Jalaja, 2022).

**Feature extraction**

The success of identification in any biometric system is based on the features obtained from biometrics (Saranya et al., 2016). Therefore, this stage is the most important stage of a biometric system (Alkababji & Mohammed, 2021). Before feature extraction, the raw biometrics must go through several fundamental steps (Saranya et al., 2016). Feature extraction is the process of creating features that can be used in selection and classification procedures. Features that help in discriminating are chosen and employed in the classification step, while features that are not chosen are excluded. Feature extraction is the most important stage since distinguishing features directly affect the efficacy of the classification task (Choras, 2007). The expression "feature extraction" refers to the procedure of obtaining a normalized ear and reducing it to a mathematical model (i.e., a feature vector) that only retains valuable and necessary information(Revina & Emmanuel, 2021).

In feature extraction, the PCA-based approach creates a face or ear area by first identifying the points and then cropping the image to a standard size around the points. The cut images are standardized to the 130x150 size specified by the PCA software. At this moment, a pixel in the ear image corresponds to a finer-grained metric region than the face image. Normalized images are masked to gray out the backdrop, leaving only the face or ears (Chang et al., 2003).

Burge and Burger (2000) proposed one of the most essential strategies utilized in ear detection. They executed the detection procedure using deformable contours and discovered that the initial contour needed user participation (Burge & Burger, 2020). As a result, the location of the ear is not completely automated. Hurley et al. (2005) used the force field approach, which does not require knowledge of the ear's location for recognition. However, this technique is only effective when a clear image of the ear is obtained (Hurley, Nixon & Carter, 2005). Yan and Bowyer (2005) used the manual detection technique described in the previous two lines. In this approach, a line is drawn at the boundary between the ear and the face, and another line runs above and below the ear (Yan & Bowyer, 2005. Cummings et al. (2010) present a strategy utilizing image ray transform to highlight ear structures with a technique that uses the helical elliptical shape of the ear for location calculation (Cummings, Nixon & Carter, 2010). Kumar et al. (2011) described a methodology involving skin segmentation and edge map recognition to locate the ear, following by the active contour method (Lankton & Tannenbaum, 2008) to determine the current location of the ear contours. Zhou et al. (2010) suggested a shape-based feature set called Histograms of Categorized Shapes (HCS) for robust 3D ear detection (Zhou, Cadavid & Abdelmottaleb, 2010; Galdamez, Arrieta & Ramon, 2014).

Briefly stated, feature extraction is the process of determining the key aspects of an image which are necessary for recognition. The purpose here is to reduce dimensionality by characterizing the complete image using only a few features (Ayoub & Oberoi, 2021).

## Conclusion

Among biometrics such as face, fingerprint, and iris, the ear has gained popularity as a biometric, alongside the face, fingerprint, and iris. The success of preprocessing and feature extraction, which are at the heart of the working idea of artificial intelligence systems that analyze ear images as biometric data, has a significant impact on the system's success.

To improve the use of ear biometrics in security investigations, more artificial intelligence-based studies should be carried out, and machine learning systems created should be provided with more data to get higher accuracy results.

## References

Abaza, A., Ross, A., Herbert, C., Harrison, M.A.F., Nixon, M.S. (2013). A survey on ear biometrics. *ACM Compt. Surv*, 45(2):22-35. doi: 10.1145/2431211.2431221

Abbas, A., Rutty, G.N. (2005). Ear piercing affects earprints: the role of ear piercing in human identification. J Forensic Sci 50(2):386–392. doi: https://doi.org/10.1520/JFS2003374

Alkababji, A.M., Mohammed, O.H. (2021). Real time ear recognition using deep learning. *TELEKOMINIKA*, 19(2):523-530. doi: 10.12928/TELKOMNIKA.v19i2.18322

Anwar, A., Ghany, K., Elmahdy, H. (2015). Human ear recognition using geometrical features extraction. Procedia Computer Science, 2015;65:529-537. doi: https://doi.org/10.1016/j.procs.2015.09.126

Ayoub, R., Oberoi, A. (2021). Study of ear biometrics based identification system using machine learning. *Int J Res Appl Sci Eng Tech (IJRASET)*, 9(7):188-194. doi: https://doi.org/10.22214/ijraset.2021.39245

Benzaoui, A., Khaldi, Y., Bouaouina, R., Amrouni, N., Alshazly, H., Ouahabi, A. (2023). A comprehensive survey on ear recognition: Databases, approaches, comparative analysis, and open challenges. Neurocomputing, 537:236-270. doi: https://doi.org/10.1016/j.neucom.2023.03.040

Booysens, A., Viriri, S. (2022). Ear biometrics using deep learning: A survey. Hindawi *Applied Computational Intelligence and Soft Computing*, doi: https://doi.org/10.1155/2022/9692690

Burge, M., Burger, W. (2000). Ear biometrics in computer vision. *ICPR*, 2:822-826. doi: 10.1109/ICPR.2000.906202

Chang, K., Bowyer, K.W., Sarkar, S., Victor, B. (2003). Comparison and combination of ear and face images in appearance-based biometrics. *IEEE Transactions on Pattern Analysis and Machine Intelligence*, 25(9):1-15. doi: 10.1109/TPAMI.2003.1227990

Chattopadhyay, P.K., Bhatia, S. (2009). Morphological examination of ear: a study of an Indian population. Leg Med. 11:S190-193. doi: https://doi.org/10.1016/j.legalmed.2009.02.057

Chhikara, K., Saini, V. (2023). Ear biometrics and morphological variation in diverse population of world. J Indian Acad For Med, 45(4):428-433. doi: 10.48165/jiafm.2023.45.4.29

Choras, R.S. (2007). Image feature extraction techniques and their applications for CBIR and biometrics systems. *Int J Bio Biomed Eng*, 1(1):6-16.

Cummings, A., Nixon, M., Carter, J. (2010). A novel ray analogy for enrolment of ear biometrics. *BTAS*, pp.1-6. doi: 10.1109/BTAS.2010.5634468

Daramola, S.A., Oluwaninyo, O.D. (2011). Automatic ear recognition system using back propagation neural network. Int J Video Image Process Netw Secur 11(1):28–32

Dargan, S., Kumar, M. (2019). A comprehensive survey on the biometric recognition systems based on physiological and behavioral modalities. Expert Systems with Applications, 143:113114. doi: 10.1016/j.eswa.2019.113114

Dhanda, V., Badhan, J.S., Garg, R.K. (2011). Studies on the development of latent ear prints and their significance in personal identification. Problems of For Sci. 138:285-295.

Dodge, A., Mounsef, J., Karam, L. (2018). Unconstrained ear recognition using deep neural networks. *IET Biom*. 7(3):207 214. doi: 10.1049/iet-bmt.2017.0208

Ebanesar, T., Bibin, A.D., Jalaja, J. (2022). Human ear recognition using convolutional neural network. *J Positive School Psychology*, 6(4):8182-8190.

Fakorede, S.T., Adekoya, K.O., Fasakin, T.P., Odubambo, O., Oboh, B. (2021). Ear morphology and morphometry as potential forensic tools for identification of the Hausa, Igbo and Yoruba populations of Nigeria. Bulletin of the National Research Centre. 45:1-19. doi: https://doi.org/10.1186/s42269-021-00665-0

Galdamez, P.L., Arrieta, A.G., Ramon, M.R. (2014). Ear recognition using a hyvrid approach based on neural networks. *17th Internation Conference on Information Fusion (FUSION),* pp.1-6.

Hurley, D.J., Nixon, M.S., Carter, J.N. (2005). Force field feature extraction for ear biometrics. *Computer Vision and Image Understanding*, pp.491-512. doi: https://doi.org/10.1016/j.cviu.2004.11.001

Iyer, A.P., Karthikeyan, J., Khan, R.H., Binu, P.M. (2020). An analysis of artificial intelligence in biometrics-the next level of security. *Journal of Critical Reviews*, 7(1):571-576. doi: http://dx.doi.org/10.31838/jcr.07.01.110

Jain, A.K., Ross, A., Pankanti, S. (2006). Biometrics: A tool for information security. *IEEE Trans. Inf. For. Security*, 1(2):125-143. doi: 10.1109/TIFS.2006.873653

Khobragade, S., Mor, D.D., Chhabra, A. (2015). A method of ear feature extraction for ear biometrics using MATLAB, *2015 Annual IEEE India Conference (INDICON)*, New Delhi, India, (pp. 1-5). doi: 10.1109/INDICON.2015.7443344.

Krishan, K., Kanchan, T., Thakur, S. (2019). A study of morphological variations of the human ear for its applications in personal identification. Egypt J Forensic Sci 9:6. doi: https://doi.org/10.1186/s41935-019-0111-0

Kumar, A., Hanmandlu, M., Kuldeep, M., Gupta, H.M. (2011). Automatic ear detection for online biometric applications. *NCVPRIPG*. doi: 10.1109/NCVPRIPG.2011.69

Kumar, P., Singla, A. (2013). Ear biometric: sex, bilateral and ethnic differences among brahmin and Yadav communities of Bundelkhand region using PCA technique. Int J Scientific & Eng Res. 4:799-805.

Lankton, S., Tannenbaum, A. (2008). Localizing region-based active contours. *IEEE Transactions on Image Processing*, 17(11):2029-2039. doi: 10.1109/TIP.2008.2004611

Liang, Y., Samtani, S., Yu, Z. (2020). Behavioral biometrics for continuous authentication in the Internet-of-Things era: An artificial intelligence persperctive. *IEEE Internet of Things Journal,* 7(9):9128-9143. doi: 10.1109/JIOT.2020.3004077

Meng, D., Nixon, M.S. (2021). On distinctiveness and symmetry in ear biometrics. *IEEE Transactions on Biometrics, Behavior, and Identity Science*, 3(2):155-165. doi: 10.1109/TBIOM.2021.3058562

Minae, S., Abdolrashidi, A., Su, H., Bennamoun, M., Zhang, D. (2023). Biometrics recognition using deep learning: A survey. *Artificial Intelligence Review*, 56:8647-8695. doi: https://doi.org/10.1007/s10462-022-10237-x

Murgod, V., Angadi, P., Halilkerimath, S., Kale, A. (2013). Anthropometric study of the external ear and its applicability in sex identification: assessed in an Indian sample. Aust J For Sci. 45(4):431-444. doi: https://doi.org/10.1080/00450618.2013.767374

Nikosa, S., Meena, H.K. (2020). Ear-biometrics for human identification. *2020 Advanced Computing and Communication Technologies for High Performance Applications*, Cochin, India. doi: 10.1109/ACCTHPA49271.2020.9213190

Ogoun, T.R. (2022). Biometrics of the external ear of the Ijaws in Bayelsa State, Nigeria. Int J Health Sci and Research, 12(7):241-245. doi: https://doi.org/10.52403/ijhsr.20220735

Omara, I., Wu, X., Zhang, H., Du, Y., Zuo, W. (2017). Learning pairwise SVM on deep features for ear recognition. *IET Biometrics*, 7(6):341-346. doi: https://doi.org/10.1049/iet-bmt.2017.0087

Ometov, A., Petrov, V., Bezzateev, S., Andreev, S., Koucheryavy, Y., Gerla, M. (2019). Challenges of multi-factor authentication for securing advanced loT applications. *IEEE Netw.*, 33(2):82-88. doi: 10.1109/MNET.2019.1800240

Rahman, M., Islam, R., Bhuiyan, N.I., Ahmed, B., Islam, A. (2007). Person identification using ear biometrics. Int J Comput Internet Manag 15(2):1–8

Rani, D., Krishan, K., Sahani, R., Shrestha, R., Kanchan, T. (2022). Characteristic features of ear and ear-prints in forensic identification. Journal of Craniofacial Surgery, 33(4):1093-1098. doi: 10.1097/SCS.0000000000008056

Revina, I., Emmanuel, W. (2021). A survey on human face expression recognition techniques. *J King Saud University-Computer and Information Sciences*, 33(6):619-628. doi: 10.1016/j.jksuci.2018.09.002

Rubio, O., Galera, V., Alonso, M.C. (2015). Anthropological study of ear tubercles in a Spanish sample. Homo. 66:343-356.

Sforza, C., Grandi, G., Binelli, M., Tommasi, D., Rosati, R., Ferrario, V. (2009). Age-and sex-related changes in the normal human ear. *Forensic Sci Int*, 187(1-3):110-111. doi: 10.1016/j.forsciint.2009.02.019

Sharkas, M. (2022). Ear recognition with ensemble classifiers; A deep learning approach. *Multmedia Tools and Applications*, 81:43919-43945. doi: https://doi.org/10.1007/s11042-022-13252-w

Swift, B., Rutty, G.N. (2003). The human ear: its role in forensic practice. J Forensic Sci 48(1):153–160

Verma, K., Bhawana, J., Kumar, V. (2014). Morphological variations of ear for individual identification in forensic cases: a study of Indian population. Res J For Sci. 2(1):1-8.

Verma, P., Sandhu, H.K., Verma, K.G., Goyal, S., Sudan, M., Ladgotra, A. Morphological variations and biometrics of ear: An aid to personal identification. J Clin Diagn Res. 10(5):ZC138-ZC142. doi: 10.7860/JCDR/2016/18265.7876

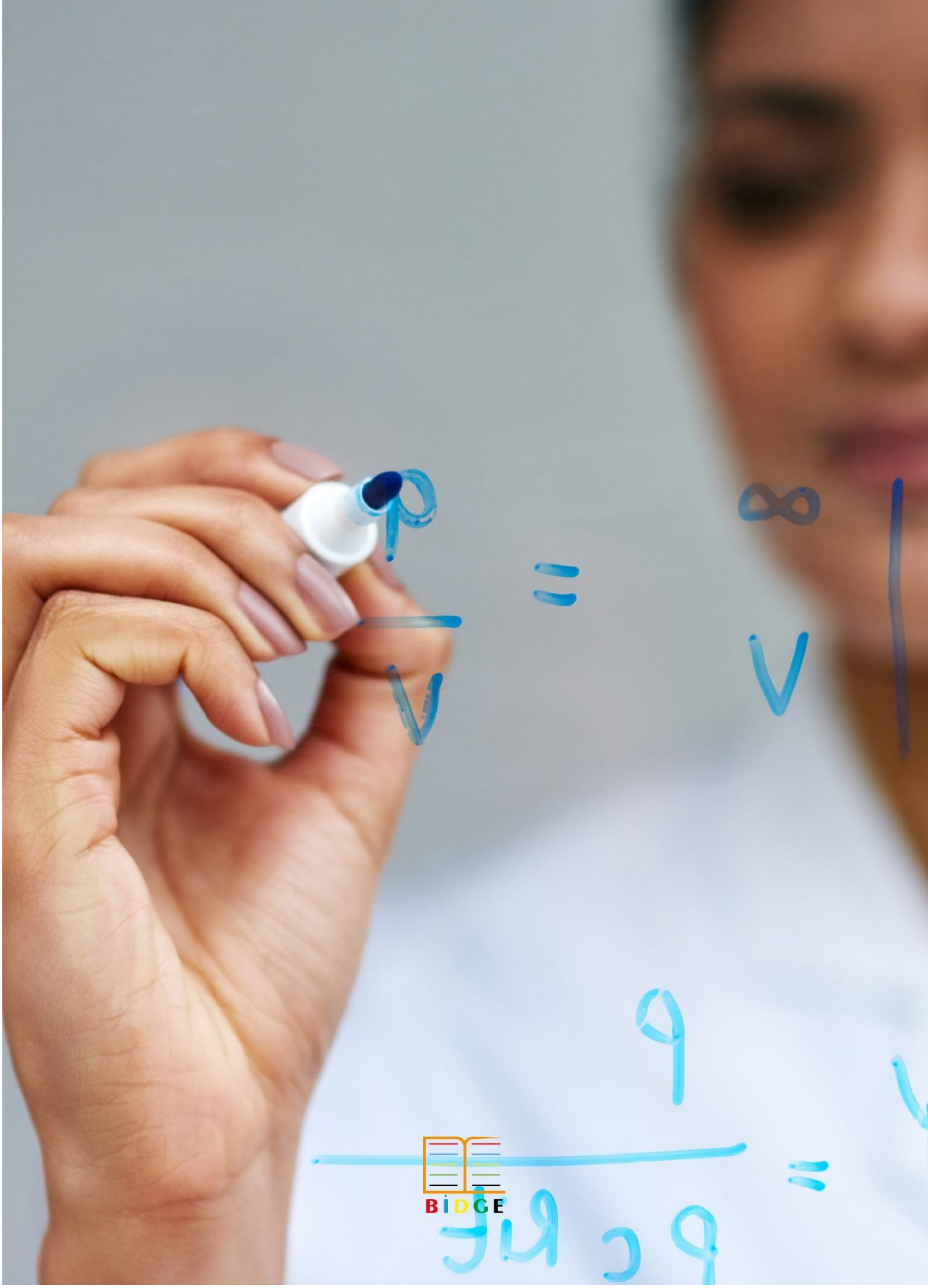
Victor, B., Bowyer, K.W., Sarkar, S. (2002). An evaluation of face and ear biometrics. Proc Int Conf Pattern Recog 1:429–432

Yan, P., Bowyer, K.W. (2005). Empirical evaluation of advanced ear bio metrics. In: Proceedings of International Conference on Computer Vision and Pattern Recognition (workshop), 3:41-48. doi: 10.1109/CVPR.2005.450

Yoga, S., Valaih, J., Rangdhol, V., Vandana, S., Paulose, S., Kavya, L. (2017). Assessment of age changes and gender differences based on anthropometric measurements of ear: A cross-sectional study. J Advanced Clinical Res Insights, 4(4):92-95. doi: 10.15713/ins.jcri.167

Zhang, Y., Mu, Z. (2017). Ear detection under uncontrolled conditions with multiple scale faster region-based convolutional neural networks. Symmetry, 9(4):53. doi: https://doi.org/10.3390/sym9040053

Zhou, J., Cadavid, S., Abdelmottaleb, M. (2010). Histograms of categorized shapes for 3D ear detection. *BTAS*, Washington-DC, USA. doi: 10.1109/BTAS.2010.5634512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