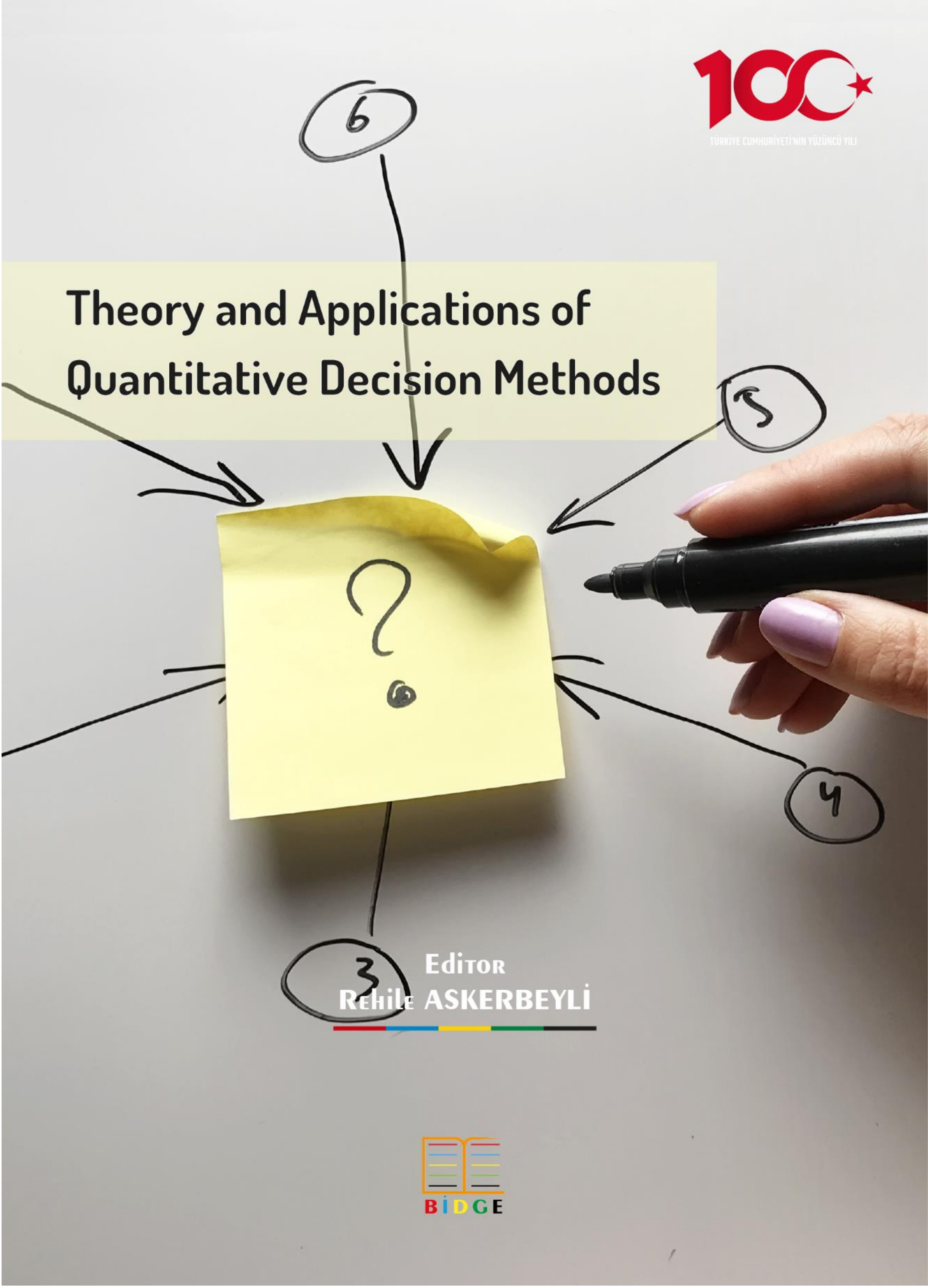
Theory and Applications of Quantitative Decision Methods



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PREFACE

Quantitative decision methods are essential for social sciences. This book, divided into four chapters, examines them from theoretical and practical aspects. The first chapter is authored by Furkan Göktaş and entitled "Comparative Analysis of the Optimization-Based MCDM Methods (NM-TOPSIS and U-PES)". This chapter focuses on two of the multi-criteria decision-making (MCDM) methods, U-PES and NM-TOPSIS, which are used to solve MCDM problems. The second chapter is authored by Oya Önalan and entitled "Quantitative Techniques for Decision-Making", where the author briefly reviews quantitative decision methods. The third chapter is authored by Canan Yıldıran and entitled "Comparing the Leadership Types in terms of Entrepreneurship: An MCDM Approach based on AI." This chapter examines the relationship between leadership types and entrepreneurship using MCDM. The fourth chapter is authored by Rehile Askerbeyli and Levent Ünalan and entitled "Application of Fuzzy Linear Programming to the Turkish Cement Sector" consists of two parts. Part 1 describes fuzzy logic theory and fuzzy linear programming. Part 2 focuses on solving the optimal distribution and transportation problem of cement produced in factories operating in the Turkish cement industry to demand points using fuzzy linear programming. This book aims to provide novel perspectives for quantitative decision methods. We hope that this book is beneficial for academicians and practitioners.

Editor

Prof. Dr. Rehile ASKERBEYLİ

Content

CHAPTER I

Comparative Analysis of the Optimization-Based MCDM Methods (NM-TOPSIS and U-PES)

Furkan GÖKTAŞ[1]

1. Introduction

Multi-criteria decision-making (MCDM) problems can be represented with a decision matrix where each column corresponds to a criterion, and each row corresponds to an alternative (Taherdoost & Madanchian, 2023). Many MCDM methods are used to solve these types of problems. Due to their objectivity, this chapter focuses on two of them (U-PES and NM-TOPSIS).

U-PES is the generalization of PES for the uncertain criteria weights (Göktaş & Güçlü, 2024b). PES combines three elementary MCDM methods: the maximin rule, the weighted sum method, and the maximax rule (Göktaş & Güçlü, 2024a). The maximin rule reflects the pessimistic point of view. The maximax rule reflects the optimistic point of view. The weighted sum method reflects a more

[1] Asst. Prof. Dr. Furkan Göktaş, Karabuk University, Faculty of Business, Department of Business Administration, Karabük/Turkey, Orcid: 0000-0001-9291-3912, furkangoktas@karabuk.edu.tr

rational point of view (Moghaddam et al., 2011; Vaidogas et al., 2007).

NM-TOPSIS is a variant of TOPSIS, a widely used MCDM method. TOPSIS has been popular since it is simple and has a strong mathematical background. On the other hand, the issue of determining criteria weights is also a matter of criticism for TOPSIS. Because the results obtained with TOPSIS depend significantly on the criteria weights (Bouslah et al., 2023). NM-TOPSIS determines the criteria weight vector inherently. Thus, it resolves the criteria weighting problem (Göktaş, 2024).

Although U-PES and NM-TOPSIS originate from different MCDM methods, they have a few similarities. Firstly, they are objective methods. That is, their results do not change for different decision-makers. Secondly, they depend on a convex optimization problem. U-PES uniquely gives the alternatives' priority vector, whereas NM-TOPSIS may sometimes give multiple optimal solutions. On the other hand, Tikhanov's regularized problem can be used to eliminate this deficiency (Göktaş, 2024).

The rest of the chapter is organized as follows. Section 2.1 gives the theory of U-PES, whereas Section 2.2 gives the theory of NM-TOPSIS. Section 3 compares these methods for the decision matrices given by Göktaş and Güçlü (2024b) or Göktaş (2024) separately. Section 4 concludes the chapter.

2. Methods

2.1. U-PES

Let $A=(a_{ij})$ be the crisp decision matrix in Equation 1, where a_{ij} is its i^{th} row - j^{th} column element.

$$A = \begin{pmatrix} a_{11} & \cdots & a_{1n} \\ \vdots & \ddots & \vdots \\ a_{m1} & \cdots & a_{mn} \end{pmatrix} \tag{1}$$

The steps of U-PES are as below (Göktaş & Güçlü, 2024a).

Step 1: The decision matrix (A) in Equation 1 is formed.

Step 2: The normalized decision matrix B=(b_{ij}) is formed using the ratio-based normalization in Equation 2, where b_{ij} is in [0,1]. α_j is the worst value for the jth criterion (or equivalently the jth column of A). β_j is the best value for the jth criterion (or equivalently the jth column of A). A and B matrices have m rows and n columns.

$$b_{ij} = \frac{|a_{ij} - \alpha_j|}{|\beta_j - \alpha_j|}, \; \textit{for all } i, j \tag{2}$$

Step 3: The security level of the ith alternative ($B_{i,1}$) is calculated using Equation 3.

$$B_{i,1} := \min_j b_{ij}, \; \textit{for all } i \tag{3}$$

The optimism level of the ith alternative ($B_{i,3}$) is calculated using Equation 4.

$$B_{i,3} := \max_j b_{ij}, \; \textit{for all } i \tag{4}$$

Step 4: The possibilistic variance vector (v) is formed using Equation 5.

$$v = \begin{pmatrix} \left(\frac{1 - B_{i,1}}{6}\right)^2 \\ \vdots \\ \left(\frac{1 - B_{m,1}}{6}\right)^2 \end{pmatrix} \tag{5}$$

Step 5: The scenario-based possibilistic mean matrix (M) is formed using Equation 6.

$$M = \begin{pmatrix} \left(\frac{B_{1,1} + 2b_{11} + B_{1,3}}{4}\right) & \cdots & \left(\frac{B_{1,1} + 2b_{1n} + B_{1,3}}{4}\right) \\ \vdots & \ddots & \vdots \\ \left(\frac{B_{m,1} + 2b_{m1} + B_{m,3}}{4}\right) & \cdots & \left(\frac{B_{m,1} + 2b_{mn} + B_{m,3}}{4}\right) \end{pmatrix} \tag{6}$$

Remark: U-PES determines the alternative's priority vector (p) as the standardized optimal solution (w^*) of Equation 7 for the w=(w_i) vector, where w corresponds to a resource allocation plan, and y is a variable associated with the worst-case scenario. (The standardization means that the vector is divided by the sum of its elements). It can be solved using CVX, which is a package for convex optimization problems (Grant & Boyd, 2008).

$$\max\ y - \frac{1}{2}\sum_{i=1}^{m} w_i^2 \left(\frac{1-B_{i,1}}{6}\right)^2$$
$$s.t.\ y \le \sum_{i=1}^{m} w_i \left(\frac{B_{i,1}+2b_{ij}+B_{i,3}}{4}\right),\ for\ all\ j \tag{7}$$

Step 6: Equation 7 is uniquely solved using an optimization solver. The CVX code is given in Equation 8. The standardized optimal solution (w^*) is taken as the alternative's priority vector (p). The criteria weight vector is objectively determined using the nonnegative dual optimal vector $\lambda=(\lambda_j)$. (The sum of the λ_j values equals 1.)

$$\begin{aligned}
&cvx_solver\ mosek\\
&cvx_begin\\
&variables\ w(m)\ y;\\
&dual\ variable\ \lambda;\\
&\quad maximize\left(y-0.5*transpose(w)*diag(v)*w\right);\\
&\quad subject\ to\\
&\qquad \lambda:\ y*ones(n,1)-transpose(M)*w <=\ zeros(n,1);\\
&cvx_end
\end{aligned} \tag{8}$$

Step 7: The priority vector $p=(p_i)$ is used for resource allocation and/or ranking the alternatives.

2.2. NM-TOPSIS

NM-TOPSIS is a TOPSIS variant based on norm (distance) minimization. It determines the criteria weight vector by minimizing the squared sum of the Euclidean distances of alternatives to the negative ideal solution. Then, it determines the alternative's priority vector (p) by minimizing the Euclidean distance to the positive ideal solution (Göktaş, 2024). The first two steps of NM-TOPSIS are identical to the first two steps of U-PES. The rest of the steps of NM-TOPSIS are as below.

Step 3: The criteria weight vector $\lambda=(\lambda_j)$ is determined using Equation 9.

$$d_j := \sum_{i=1}^{m} b_{ij}^2 \rightarrow \lambda_j = \frac{1/d_j}{\sum_{j=1}^{n} 1/d_j}, \textit{ for all } j \tag{9}$$

Step 4: The weighted normalized decision matrix $C=(c_{ij})$ is formed using Equation 10.

$$c_{ij} = \lambda_j b_{ij}, \textit{ for all } i, j \tag{10}$$

Step 5: Equation 11 is solved for the w vector. The CVX code is given in Göktaş (2024). The alternatives's priority vector (p) is taken as equal to the optimal weight vector (w^*). Here, C^T is the transpose matrix of the C matrix. The objective function is the 2-norm of the (λ-C^Tw) vector.

$$\begin{aligned} &\min \left\| \lambda - C^T w \right\|_2 \\ &k.a. \sum_{i=1}^{m} w_i = 1 \\ &\quad w_i \geq 0, \forall i \end{aligned} \tag{11}$$

Remark: If $H:=2CC^T$ is a positive-definite matrix, then Equation 11 has a unique optimal solution. If it is a positive semi-definite matrix, then Tikhonov's regularized problem can be used to obtain the optimal solution closest to the origin approximately. For detailed information, see Göktaş (2024).

Step 6: The priority vector $p=(p_i)$ is used for resource allocation and/or ranking the alternatives.

3. Results and Discussion

In this section, we compare U-PES and NM-TOPSIS using two examples in the literature. The first example is given by Göktaş (2024), where the decision matrix as in Table 1. Clearly, there are five alternatives and six criteria. All criteria are utility criteria.

Table 1: The decision matrix (A) for Example I.

	C1	C2	C3	C4	C5	C6
A1	96.04	94.22	42.89	82.57	96.90	87.79
A2	93.34	96.81	60.18	45.09	95.85	47.91
A3	84.56	99.68	49.45	26.47	96.61	71.92
A4	100.00	100.00	46.39	63.67	97.01	74.07
A5	94.42	94.85	57.21	43.10	95.48	34.59

Göktaş (2024) gives the steps of NM-TOPSIS for Example I. The steps of U-PES are as below.

Step 1: The decision matrix is as in Table 1.

Step 2: We form the normalized decision matrix as in Table 2 using Equation 2.

Table 2: The normalized decision matrix (B) for Example I.

	C1	C2	C3	C4	C5	C6
A1	0.7437	0.0000	0.0000	1.0000	0.9266	1.0000
A2	0.5683	0.4484	1.0000	0.3318	0.2390	0.2504
A3	0.0000	0.9440	0.3796	0.0000	0.7411	0.7018
A4	1.0000	1.0000	0.2022	0.6631	1.0000	0.7420
A5	0.6386	0.1083	0.8285	0.2963	0.0000	0.0000

Step 3: We find the security level of the i^{th} alternative ($B_{i,1}$) as in Table 3 using Equation 3, whereas we find the optimism level of the i^{th} alternative ($B_{i,3}$) using Equation 4.

Table 3: The security and optimism levels for Example I.

	$B_{i,1}$	$B_{i,3}$
A1	0.0000	1.0000
A2	0.2390	1.0000
A3	0.0000	0.9440
A4	0.2022	1.0000
A5	0.0000	0.8285

Step 4: We form the possibilistic variance vector (v) as in Table 4 using Equation 5.

Table 4: The possibilistic variance vector for Example I.

	Value
A1	0.0278
A2	0.0161
A3	0.0278
A4	0.0177
A5	0.0278

Step 5: We form the scenario-based possibilistic mean matrix (M) as in Table 5 using Equation 6, where the weight of C1 equals 1 in Scenario I (S1). Similar information is also valid for S2, S3, S4, S5, and S6.

Table 5: The possibilistic mean matrix for Example I.

	S1	S2	S3	S4	S5	S6
A1	0.6821	0.6620	0.6833	0.6045	0.7143	0.4762
A2	0.9475	0.9475	0.9475	0.9475	0.9475	0.7963
A3	0.6535	0.6085	0.6650	0.5816	0.7402	0.5378
A4	0.7639	0.7477	0.7572	0.7147	0.7785	0.5916
A5	0.5000	0.5000	0.5000	0.5000	0.5000	0.7500

Step 6: We find the criteria weight vector (λ) as in Table 6, by implementing the CVX code given in Equation 8. That is, U-PES objectively determines the weight of C4 equals %100. The weights of other criteria equal 0 since the optimal resource allocation plan (p) is relatively stronger for these criteria rather than C4.

Table 6: The criteria weight vectors (λ) for Example I.

	U-PES	NM-TOPSIS
C1	0.0000	0.1490
C2	0.0000	0.1617
C3	0.0000	0.1818
C4	1.0000	0.2078
C5	0.0000	0.1381
C6	0.0000	0.1616

We find the alternative’s priority vector (p) as in Table 7, by implementing the CVX code given in Equation 8.

Table 7: The priority vectors (p) for Example I.

	U-PES	NM-TOPSIS
A1	0.1539	0.0076
A2	0.3938	0.2332
A3	0.1213	0.0000
A4	0.2308	0.7606
A5	0.1002	0.0000

Step 7: U-PES ranks the alternatives as A2>A4>A1>A3>A5 based on U-PES using the priority values.

NM-TOPSIS ranks the alternatives as A4>A2>A1>A3=A5. NM-TOPSIS and U-PES give similar rankings since the Spearman rank correlation equals 0.872, which is quite high (Hair et al., 2007).

The second example is given by Göktaş and Güçlü (2024b), where the decision matrix as in Table 8. There are five alternatives and six criteria. C1, C2, C3, and C4 are utility criteria, whereas C5 and C6 are cost criteria. We skip the steps of U-PES. The steps of NM-TOPSIS are as below.

Table 8: The decision matrix (A) for Example II.

	C1	C2	C3	C4	C5	C6
A1	0.0133	-0.0025	0.3148	0.0127	0.0316	0.7844
A2	0.0144	0.0333	0.3600	0.0161	0.0307	0.6840
A3	0.0117	-0.0518	0.2586	0.0110	0.0323	0.7573
A4	0.0135	0.0052	0.3169	0.0138	0.0321	0.7401
A5	0.0081	-0.1060	0.0965	0.0090	0.0495	0.5301

Step 1: The decision matrix is as in Table 8.

Step 2: We form the normalized decision matrix as in Table 9 using Equation 2.

Table 9: The normalized decision matrix (A) for Example II.

	C1	C2	C3	C4	C5	C6
A1	0.8238	0.7431	0.8285	0.5134	0.9524	0.0000
A2	1.0000	1.0000	1.0000	1.0000	1.0000	0.3950
A3	0.5690	0.3893	0.6152	0.2814	0.9160	0.1064
A4	0.8633	0.7986	0.8364	0.6665	0.9219	0.1743
A5	0.0000	0.0000	0.0000	0.0000	0.0000	1.0000

Step 3: We find the criteria weight vector (λ) as in Table 10 using Equation 9. Here, the weight of C6 equals %29.55.

Table 10: The criteria weight vectors (λ) for Example II.

	U-PES	NM-TOPSIS
C1	0.0000	0.1288
C2	0.0000	0.1512
C3	0.0000	0.1280
C4	0.0000	0.1981
C5	0.0000	0.0984
C6	1.0000	0.2955

Step 4: We form the weighted normalized decision matrix (C) as in Table 11 using Equation 10.

Table 11: The C matrix for Example II.

	C1	C2	C3	C4	C5	C6
A1	0.1061	0.1123	0.1061	0.1017	0.0937	0.0000
A2	0.1288	0.1512	0.1280	0.1981	0.0984	0.1167
A3	0.0733	0.0588	0.0788	0.0557	0.0902	0.0314
A4	0.1112	0.1207	0.1071	0.1320	0.0907	0.0515
A5	0.0000	0.0000	0.0000	0.0000	0.0000	0.2955

Step 5: We uniquely find the priority vector (p) as in Table 12 by solving Equation 11.

Table 12: The priority vectors (p) for Example II.

	U-PES	NM-TOPSIS
A1	0.0964	0.0000
A2	0.4401	0.7662
A3	0.1363	0.0000
A4	0.1756	0.0000
A5	0.1517	0.2338

Step 6: Using the priority values, NM-TOPSIS ranks the alternatives as A2>A5>A1=A3=A4.

U-PES ranks the alternatives as A2>A4>A5>A3>A1. The Spearman rank correlation between these rankings equals 0.671. This corresponds to the moderate positive relationship since it is between 0.41-0.70 (Hair et al., 2007).

Wıth the help of the examples, we may list the differences between U-PES and NM-TOPSIS as below.

- U-PES tends to ignore some criteria due to a Karush-Kuhn-Tucker (KKT) condition, whereas NM-TOPSIS considers all criteria.
- U-PES gives the all-rankings, whereas NM-TOPSIS may not differentiate the low-ranking alternatives.
- U-PES simultaneously determines the criteria weight vector and the alternative's priority vector, whereas NM-TOPSIS determines them in order.
- U-PES considers the security levels, the optimism levels, and the normalized decision matrix, whereas NM-TOPSIS only considers the normalized decision matrix.

4. Conclusions

This chapter makes a comparative analysis of U-PES and NM-TOPSIS using two examples from the literature. In both examples, we find different criteria weight vectors and priority vectors. On the other hand, they give similar rankings for Example 1. The similarity between the rankings for Example II is at the medium level. Since these methods have different perspectives, these differences are not surprising. We remark that there is no superior relationship between these objective MCDM methods. If the decision-maker wants to consider the Euclidean distance to the positive (negative) ideal solution, then s/he may prefer NM-TOPSIS. If the decision-maker wants to consider the security levels and the optimism levels in addition to the normalized decision matrix, then s/he may prefer U-PES.

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CHAPTER II

Quantitative Techniques for Decision-Making

Oya ÖNALAN[1]

1. Introduction

The early 20th century saw firms utilizing statistical and mathematical techniques to enhance their operations, which led to the development of quantitative techniques in management theory. However, as governments and corporations employed statistical analysis to boost productivity and efficiency, quantitative techniques in management were widely recognized during World War II (Suzuki, 1967).

Quantitative techniques are becoming essential components of contemporary management theory and practice. Introducing quantitative techniques into management has aided companies in increasing production, profitability, and efficiency by offering a methodical and impartial approach to decision-making (Nkuda, 2020).

[1] Asst. Prof. Dr. Oya Önalan, Karabuk University, Faculty of Business, Department of Business Administration, Karabük/Turkey, Orcid: 0000-0002-4169-8789, oyaonalan@karabuk.edu.tr

This chapter briefly reviews the quantitative techniques for decision-making. The rest of the chapter is organized as follows. Section 2 presents the importance of decision-making for management. Section 3 presents the quantitative and qualitative data. Section 4 presents the importance of quantitative techniques. Section 5 presents the classification of quantitative techniques. Section 6 presents the applications of quantitative techniques. Section 7 presents the quantitative data collection. Section 8 concludes the chapter.

2. Importance of Decision-Making for Management

The ever-intricate business environment in which organizations operate has left managers grappling with matters ranging from the relatively insignificant to the strategically significant. Consequently, the information requirements of managers have grown increasingly intricate, challenging, and demanding. A manager needs more time to evaluate, analyze, and respond to an opportunity or problem. The decision-making process is difficult since many variables and a need for more trustworthy information exist. Managers can, however, make wise choices that will support the success of their organizations if they choose a reasonable strategy (Meduru, 2023).

Also, managers must be ready to change their minds when new information becomes available. They should stay flexible and agile and be prepared to change course when circumstances warrant. Managers must make decisions quickly and, presumably, appropriately, with the assistance of their information systems. According to Meduru (2023), quantitative techniques are essential to commercial decision-making because they enable organizations to objectively and methodically analyze and evaluate vast amounts of data.

A decision is an option selected from a list of two or more options. Making decisions in the face of uncertainty involves deciding between two or more options when it is unclear how those choices will turn out. Jaques (1994) says, *"If you have made a decision entirely based on factual information, you have not made a decision; it was made for you by the facts."*

Effective decision-making is an important part of maximizing effectiveness and efficiency in the workplace. Hence, decision-making became the heart of the management/administrative process (Mann, 1976).

Modern managers resort to quantitative techniques to facilitate their managerial decision-making since most organizational decisions are taken in an atmosphere of risks and uncertainties. According to Lucey (1996), quantitative techniques represent a "*modern science attack on complex problems arising in the direction and management of large systems of men, machines, materials and money (4Ms) in industry, business, and government."* In other words, quantitative techniques are "*tools used to help management determine its policy and action scientifically*."

An essential managerial skill is allocating and utilizing resources to effectively and efficiently achieve optimal performance, especially in complexity and dynamism; decision-making is based on intuition with a quantitative base that is reasonably accepted and practiced in achieving organizational goals. It is pertinent for managers to enhance their decision-making competence through experiential and scientific knowledge acquisition (Anderson et al., 1994). With a thorough understanding of the underlying assumptions, the goals of the analysis, the compromises that the model makes with reality, and how to adapt the model's results to changing environments and non-tangible factors, managers can use analytical tools appropriately.

Rumsfeld (2024) says, *‘‘As we know, there are known knowns – these are things we know we know. We also know there are known unknowns – we know there are some things we do not know, but there are also unknown unknowns – the ones we do not know, we do not know.... It is the latter category that tends to be the difficult one.’’*

3. Quantitative and Qualitative Data

There are two main categories (quantitative and qualitative) into which research and its data can be divided. Gathering enough data and information is necessary to analyze both sorts of data. Measures of values or counts expressed as numbers are quantitative data, information about numerical variables, such as quantity, frequency, or number. A name, symbol, or numeric code might denote measures of 'types' qualitative data. Data about categorical variables, such as what kind, are known as qualitative data. Information gathered regarding a numerical variable is invariably quantitative, while details about a categorical variable are invariably qualitative (Stevens, 2023).

Qualitative data is individualized, subjective, and analyzed by classifying the information into themes and categories. It helps us comprehend the why, how, or what transpired behind behaviors. Qualitative research aims to disclose the intricate details, motivations, and hidden stories that quantitative analysis cannot provide. To assist in explaining behaviors and patterns, it delves deeply into individual experiences, thoughts, and feelings rather than analyzing statistics. In qualitative research, words, images, and observations are more important than numbers. You can get rich, unstructured data through focus groups, open-ended surveys, and interviews (LeCompte, 2000).

Qualitative research is useful in identifying patterns in ideas , beliefs, or viewpoints. It is a far more individualized study area and frequently lacks objective facts, like statistical data. Rather, the goal is to investigate the core of beliefs. Qualitative decision-making is useful in business for several tasks, including customer service, product creation, and market research (Ugwu & Eze, 2023)

Any information that can be expressed as numbers is called quantitative data. The quantitative data is both definitive and objective as it is numerical. Something is considered quantitative if it is measurable and countable. Moreover, charts and graphs may represent quantitative data graphically because they are highly ordered for mathematical analysis. In calculations, quantitative data provides the number, amount, or frequency. It is fixed, universal, and subjected to statistical examination (Timonera, 2024). Quantitative research gathers numerical data that can be converted into statistics for making the right decisions.

4. Importance of Quantitative Techniques

Quantitative techniques play a crucial role in the marketing domain and consumer behavior analysis. They aid in market research, action analysis, and overall marketing strategy optimization. It can also be used for demand forecasting, pricing research, consumer segmentation, and improving marketing campaigns.

According to Stanton et al. (2019), employing quantitative techniques gives organizations the strategic benefit of discovering opportunities, minimizing risks, and securing a competitive edge in a dynamic market since they are based on rigorous research and empirical data. Fundamentally, these methods are the cornerstone of a sound, data-driven decision-making procedure. In the past, managers relied on their expertise to make decisions. However, as data technology has advanced, quantitative analysis is better for well-informed decisions.

Organizations can proactively coordinate strategy and anticipate shifts using quantitative techniques. They may identify data trends, patterns, and correlations using quantitative analysis, laying a strong basis for strategic planning and management and are essential for most organizations because they provide unbiased, numerical insights that enable well-informed decision-making (Bagshaw & Nissi, 2019).

The twenty-first-century advances in communications, informatics systems, and knowledge management, along with the demands placed on business organizations to stay informed of these developments, have rendered the conventional approaches to administrative decision-making impractical (Mohajan, 2016).

Making decisions is essential when it comes to strategic planning. The success of an organization can be determined by its ability to make data-driven and well-informed decisions. Numerous quantitative methods are frequently employed in many fields of strategic planning, including:

Statistical Process Control and Six Sigma approaches are commonly used in quantitative techniques for quality control and to monitor and improve operations. The decision tree technique allows for scientific decision-making in risk and unclear situations by honing executive judgment through systematically investigating the problem (Magee, 1964).

The linear programming technique is highly helpful for determining the best assignments, production scheduling, and the optimal distribution of scarce resources. A manager can reduce waiting and servicing expenses using the Waiting Line Theory to solve cost-minimization issues (Heizer & Render, 2014).

5. Classification of Quantitative Techniques

Quantitative approaches can be broadly divided into two categories: mathematical and statistical. Some common ones are as follows: Mathematical quantitative techniques are methods that combine the use of quantitative data with mathematical principles (Taylor, n.d.). They include a wide range of tools.

Linear programming: The method of linear programming is used to determine how to achieve a goal best while adhering to specified restrictions. In decision-making circumstances, mathematical optimization is applied to determine the optimal result. Businesses may maximize earnings, optimize production processes, and allocate resources efficiently using linear programming models. They accomplish this by taking goals and limitations into account at the same time. This method aids in making the most of an object with constrained resources (Heizer & Render, 2014) *Non-Linear programming*: This programming technique determines the best answer to a problem where some or all the variables are not linear.

Game theory: Managers frequently reserve the use of certain quantitative techniques for decisions involving their competitors in the market. The "game theory" discipline of thought seeks to determine the best action in every situation. It employs thought experiments and quantitative techniques, consistently selecting the best course of action in a competitive setting (Kelly, 2023)

Techniques employed in statistical research related to a particular phenomenon are known as *statistical techniques*. They comprise every statistical technique, from data collection to data analysis. It covers a wide range of statistical concepts and tools.

Probability decision theory: This theory deals with how uncertainty is measured. Business organization managers make decisions in the face of risks, ambiguity, and certainty. The probability decision theory aims to assist managers in identifying potential outcomes that may be connected to favorable results. Mutually exclusive occurrences are scenarios in which the occurrence of one event is contingent upon the occurrence of another (Taghavifard et al., 2009). However, the scenario depicts independent events if one event's occurrence does not influence the occurrence of another.

Analysis of payback: Applications for quantitative approaches are widely used. The payback analysis is mostly used in project appraisal, where it helps to ascertain how long a given project can recoup its investment costs based on the project's anticipated cash inflow profile (Ani, 2015).

Regression analysis: Many people utilize regression analysis daily in their professional lives since it is such a helpful tool. Dependent and independent variables are the two data sets used in regression analysis. It looks at the relationships between variables to forecast and make wise judgments. Regression analysis reveals which factors are most crucial, which ones should be ignored, and how each factor interacts with the others. It is frequently used to predict price strategies and do market research because it offers important insights into the variables affecting results (Taylor, n.d.).

Regression analysis is useful for determining the potential influence of one variable, X, on another, Y. However, regression can also be used in business environments. It can first determine whether a dependent variable and a group of independent variables are related (Mac'ódo & D.S., 1999). *Correlation* analysis: The strength and direction of a relationship between two or more continuous variables are described by correlation (Mac'ódo & D.S., 1999). Its value varies from -1 to 1, with values closer to 1 indicating a high correlation and values closer to 0 indicating a weak correlation. There are various scenarios in business when two factors are correlated. For instance, balance between work and personal life, age, gender, education level, and performance may all impact an employee's motivation at work. Performance is additionally affected by incentives based on performance.

Testing of hypothesis: A statistical tool for assessing the validity of conclusions made from sample studies is hypothesis testing. An assumption or assertion concerning a population statistic is called a hypothesis. Because time and resources often prevent us from studying the entire population, we attempt to calculate a population statistic using a sample statistic. Data is gathered to compare the sample mean's actual value with its hypothesized value, testing the population assertion's validity. Next, we determine whether the difference is significant using probability distributions. Typically, hypotheses are expressed as null and alternate hypotheses. A null hypothesis is a situation in which no relationship or change exists in any of the variables of interest. An alternative theory suggests a substantial change, impact, or relationship between variables, which is the opposite of the null hypothesis (Massey & Miller, n.d.).

The Z test, T-test, independent sample t-test, Chi-square test, and others can be used for hypothesis testing.

Analysis of variance (ANOVA): Using the Analysis of Variance technique, we can concurrently examine the significance of differences between two or more sample means. This breaks down the overall variation into several parts corresponding to the different variation sources. This method can ascertain whether our samples come from the same population and have comparable means (Moore et al., 2013).

Simulation: It is a method of testing a model that simulates actual circumstances. Modeling by simulation is creating a digital prototype of a physical model to predict how well it will operate and under what circumstances a part could fail in real life (Chung, 2003).

Many professionals use Computer simulation modeling to grasp "what if" situations. Computer simulation modeling, utilizing software to reproduce a proposed system, can aid in designing complicated systems if changes to the real system are difficult to accomplish (Dodgson et al., 2007).

There are four categories of simulation models: Monte Carlo simulation, Discrete event simulation, System dynamic simulation, and Agent-based simulation.

Monte Carlo simulation (Stochastic modeling): It is a risk analysis technique. The Monte Carlo simulation is one stochastic model that may approximate portfolio performance by simulating the probability distributions of individual stock returns. Stochastic modeling uses random variables to predict the likelihood of distinct outcomes under certain scenarios (Mooney, 1997). This model simulates the behavior of a system over time by concentrating on the occurrence of specific events.

Discrete event simulation: It simulates how a system might act at a specific moment by focusing on the occurrence of particular events. It accurately and efficiently represents systems with few numbered events, but it is only useful for modeling systems with a few events (Law & Kelton, 2000).

System dynamics simulation: This method simulates the behavior of complex systems over time by concentrating on the interactions and feedback loops between different system components (Forrester, 1971).

Agent-Based modeling & simulation: A model that investigates the influence of an "agent" on the "system" or "environment" is known as an agent-based simulation. In agent-based models, a person, a piece of machinery, or almost anything else might be the agent. The agents' behavior in the simulation serves as guidelines for how those agents must behave within the system. Next, you examine the way the system reacts to those regulations

(Klügl & Bazzan, 2012). However, it would be best to base your rules on real-world facts to get genuine insights.

Time Series Analysis: Time series analysis is a technique used to examine data points collected over an extended period. Time series analysis lets us get insightful information from chronologically arranged data pieces. Identifying temporal trends, which reveal the underlying story of the data's progression across time, is at the core of time series analysis (Hayes, 2022). These trends provide insights that influence decision-making in various disciplines and act as crucial links between the past, present, and future.

Statistical quality control: To guarantee the high standard of goods produced, statistical quality control is employed. This tool helps identify the quality changes from assignable sources and random factors. Various control charts are used in the process of managing product quality (Nagar & Gahlot, 2022).

6. The Applications of Quantitative Techniques

"Quantitative techniques" refer to statistical and mathematical instruments used to analyze quantitative data and make smart business decisions. By assisting companies in making wise decisions, these technologies enable them to use their limited resources best.

Quantitative techniques are essential in the finance and investment sector for risk assessment, portfolio optimization, and the facilitation of well-informed investment decisions. These methods also apply to time-series analysis, revealing patterns and trends over an extended period and providing a strong basis for long-term financial planning. These methods quantify the possible effects of risks in addition to identifying them (Kenton, 2023)

Quantitative techniques are utilized in information technology and data analytics to derive significant insights from extensive datasets. These methods enable real-time decision-making improvements, which helps with process optimization and data-driven strategic initiatives. Additionally, these technologies assist companies in making the best use of their limited resources (Wason, 2024).

Applying quantitative techniques can effectively address managerial decision problems in business and industrial operations. Using quantitative tools in management frequently calls for turning intricate real-world situations into mathematical representations. Although this simplification can help decision-making, it might need to include some subtleties and background information that could affect the results. Due to their analytical and impartial methodology, quantitative techniques offer a foundation for accurate analysis of the cause-and-effect relationship and enable the measurement of business risk (Nkuda, 2020).

Managers by using quantitative techniques, can apply logical reasoning to investigate organizational challenges. Quantitative tools are crucial when determining the most profitable course of action and evaluating the relative profitability of several options. What proportion of each product should be used, which location among possibilities to pick, and how can orders be scheduled to maximize profitability in quantity and time?

7. Quantitative Data Collection

Numerical data examined statistically is referred to as quantitative data collecting and is frequently used in research techniques to find the relationships between variables. Statistical software can analyze the numerical data obtained through quantitative approaches (Rana et al., 2021).

Double-Blind or Triple-Blind trials are said to be the most effective ways to gather quantitative data that is impartial and practical, and they provide more precise results for a given serving size. There are numerous ways to collect quantitative data depending on the circumstances in which you are trying to acquire data. Using sensors and electronic surveying instruments, you can collect numerical data from inanimate objects (Taylor, n.d.).

It can be more challenging to obtain correct data when collecting quantitative data from individuals. You can get useful information from surveys and questionnaires, but the information may need to be more reliable. Many people may provide false information on a questionnaire for various reasons.

The two main types of quantitative data are *discrete and continuous*. According to Jackman (1998), discrete data has limited values, while continuous data has values that fall on a continuum and may contain fractions or decimals. Any offline or online technique that facilitates the acquisition of numerical data is a tried-and-true means of gathering quantitative data.

Probability sampling is an accurate sampling technique that allows researchers to create a probability assertion based on data randomly collected from the intended population through a random selection process.

Bhat (n.d.) says there are four important categories of probability sampling:

Basic random sampling: The targeted population is often selected to be included in the sample.

Cluster sampling: This method involves breaking up a population into smaller groups or clusters and then choosing a representative sample randomly from each cluster. This approach is employed when taking a random sample from the complete population is not feasible or cost-effective.

Systematic sampling: This method includes members of any specified demographic in the sample, but only the first unit is chosen randomly; the remaining units are selected orderly.

Stratified sampling: When constructing a sample, it enables selecting each unit from a specific subset of the intended audience. It is helpful when the researchers are picky about who they include in the sample.

Whether for qualitative or quantitative research, online questionnaires and surveys made with online survey software are essential to online data collecting. The survey aims to validate the respondents' actions and confidence.

8. Conclusions

Today's corporate world faces numerous obstacles because of social, political, economic, and technological advancements. To make well-informed judgments, managers must possess the necessary technical skills. Managers must, therefore, employ pertinent quantitative approaches and procedures that provide a range of options to select the best choice and make both successful and efficient decisions.

Using a methodical scientific approach with mathematical models, hypotheses, and assumptions for assessing and interpreting hard data to make effective and efficient decisions is referred to as quantitative methods or techniques.

Managers can make more accurate decisions thanks to quantitative methods since they allow them to make correct (qualified) decisions quickly, know how to apply and utilize mathematical models to solve problems and come to wise decisions to reach predetermined goals, such as cost reduction and profit maximization, to apply multi-variate analysis to analyze situations holistically.

Decisions made using quantitative methods are free of bias because they are objective.

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CHAPTER III

Comparing the Leadership Types in terms of Entrepreneurship: An MCDM Approach based on AI

Canan YILDIRAN[1]

1. Introduction

It can be said that as complexity and uncertainty increase, the difficulty level of existing and sustaining existence also increases. In the new world order, which consists of many dynamic factors such as rapid innovations or rapid and disruptive technological changes, there is a need for leaders and leaderships that have updated versions that enable organizations to successfully continue their existence. This variable and dynamic environment necessitates a holistic approach that involves multi-dimensional communication and interaction rather than a one-way approach in management and leadership. In an increasingly competitive environment, it is undoubtedly important for leaders to be able to manage tangible and intangible assets efficiently and strategically. The strategic role of human resources in achieving goals is accepted. The priority in

[1] Assoc. Prof. Dr. Canan Yıldıran, Karabuk University, Faculty of Business, Department of Finance and Banking, Karabük/Turkey, Orcid: 0000-0001-8245-197x, cananyildiran@karabuk.edu.tr

human resources management is to ensure high and continuous performance. Committed and motivated employees are needed for sustainable performance. At this point, leaders have very important responsibilities. One of the reasons why leaders are effective in organizational success is that they have an impact on performance. Baltacı and Balcı (2017: 32) state that leadership is seen as an important mechanism that can manage the challenges brought by the information age. In a period of rapid change and transformation, organizations will gain competitive power by supporting entrepreneurial behaviors, especially innovative thinking. Miller (1983: 770) defines an entrepreneurial firm as one that "engages in product-market innovation, undertakes somewhat risky ventures, and is the first to introduce 'proactive' innovations, beating its competitors". From this perspective, an entrepreneurially oriented organization makes strategic decisions to enter new markets. Therefore, it can be stated that an organization with an entrepreneurial orientation must support its employees with an entrepreneurial orientation.

When the literature on leadership styles is examined, many leadership styles are encountered. There are many different leadership styles other than those discussed in this study. As leadership styles become more widespread, confusion arises as to which leadership model is more effective in the desired subject/organization. The seven leadership styles selected within the scope of the research were briefly examined, and then entrepreneurial orientation was discussed.

This study aims to compare the leadership styles in terms of entrepreneurship. PES, a multi-criteria decision-making (MCDM) method, is used to achieve this aim. Data is obtained based on expert knowledge (AI chatbots). The rest of the chapter is organized as follows. Section 2.1 presents the examined leadership types. Section 2.2 presents the entrepreneurial orientation. Section 3 presents PES. Section 4 uses PES to compare the leadership types in terms of entrepreneurship. Section 5 discusses the results and concludes the chapter.

2. Conceptual Framework

2.1. Leadership Styles

Transformational Leadership: Pieterse et al. (2010: 610) state that transformational leadership is a high-level structure consisting of different components, while Al-Husseini et al. (2021) state that it is an important and effective factor in innovation and knowledge management systems. Burns (1978) states that transformational leadership occurs when followers are allowed to interact with each other to motivate each other and reach higher motivational and moral levels. The purpose of this leadership is for a leader to inspire her/his followers to be more productive and ethical. In transformational leadership, inspiration (Yukl, 2013: 323) will ensure that followers are connected to their values and ideals through tasks that have inspiring motivation and vision. While Bass and Bass (2008) state that transformational leadership can lead individuals to want to change, develop and be led; Yukl (2013) states that followers need to appeal to their moral values so that they can increase their awareness of ethical issues and mobilize their energy and resources to reshape organizations. Transformational leadership, which is generally stated to emerge in times of change and distress (Alrowwad et al. 2020: 200), also envisages resorting to professional ethics (Berkovich & Eyal, 2021). Transformational leaders encourage and motivate their followers to work beyond their own interests and identify themselves with the goals and vision of the organization (Saira et al., 2021: 132), and they use tangible rewards as well as intangible rewards (personal development, recognition, increasing self-esteem) to motivate employees (Keskes et al., 2018: 272).

Transactional Leadership: Transactional leadership involves interactions or transactions between the leader and the follower and encourages followers to contribute new ideas through rewards (Cai et al., 2023: 321). Transactional leadership, also described as leader-follower exchange, shares rewards such as salary and appreciation in return for good performance or a bad evaluation as punishment with followers (Spitzbart, 2013: 70). In other words, it can monitor the work of its followers, reward successful followers, and punish or warn followers who deviate from predetermined goals (Alwali & Alwali, 2022: 934). Transactional leadership intervenes in the business process when problems arise or procedures and standards are not met (Avolio & Bass, 2004) and uses the cost-benefit relationship to persuade followers to obtain value (Bass & Bass, 2008) and utilizes utilitarian ethics in this process (Berkovich & Eyal, 2021). Bass (1990) transactional leadership focuses on rewards and task accomplishment in the process that concerns transactions between individuals. Avolio and Bass (2004) transactional leadership clarifies the roles and tasks of followers, monitors their performance, and takes corrective action when necessary. Alwali and Alwali (2022: 934) transactional leadership prioritizes supervising, appointing, organizing, monitoring and controlling the performance of each employee and Sané and Abo (2021: 1625) guides followers by explaining roles and tasks to achieve goals. Transactional leadership involves the use of contingent rewards (and/or sanctions) to align followers' self-interests with organizational goals. Moreover, since desired rewards (and undesirable sanctions) are contingent on specific efforts or outcomes, followers' self-interest is expected to realize, guide, and sustain behaviors aimed at these efforts and outcomes (Jacobsen et al., 2022: 118). Transactional leadership motivates followers by appealing to their personal interests and thus exchanging benefits (Yukl, 2013), and for this purpose, it directs the behavior of followers (Bass & Bass, 2008), and uses concrete rewards (money and status) to motivate in this process (Keskes et al., 2018: 271), and in some cases, it may use the punishment method (Bass & Bass, 2008).

Servant Leadership: Servant leadership is defined as knowledgeable, ethical individuals who empower their followers and are sensitive to personal concerns, value growth, and can add value to organizations and the people they serve (Meuser & Smallfield, 2023), and it is accepted that it can be valid in modern organizations with highly educated and conscious followers (Zarei et al., 2022: 172). Given its focus on values, it is not only the behavior but also the general attitude towards people in an organization and the motivation to be a leader that distinguishes servant leadership from other leadership styles (van Dierendonck, 2011: 1231). The servant leader makes decisions and exhibits behaviors to protect the interests of all stakeholders (Eva et al., 2019: 128). Servant leadership advocates that long-term organizational goals will be achieved by facilitating the development and well-being of followers (Hoch et al., 2018: 507), and for this purpose, the servant leader prefers one-to-one communication to bring out the best in his followers and understand their abilities, needs, desires, goals and potential (Liden et al., 2008: 162). Servant leadership supports the development of followers (Russell & Stone, 2002) it encourages its followers to contribute to organizational goals, develops a sense of belonging, increases confidence in their work and leadership skills, and attaches importance to happiness, health, and productivity (Meuser & Smallfield, 2023). In addition, servant leadership pays attention to developing a spirit of servanthood among its followers to create value in society (Liden et al., 2008: 163). Thus, it will be able to contribute to society in the long term (Winston & Fields, 2015). In addition, servant leadership enhances the emotional well-being of followers by focusing on serving their primary needs. Thus, the personal development of the followers will be ensured through the positive effects that arise in relation to their feelings of emotional well-being (Jiménez-Estévez et al., 2023). Mcquade et al. (2021) state that the skills required for servant leadership are trust, communication, empathy and listening. Trust develops in the superior-subordinate relationship when followers perceive the servant leader’s decisions and actions as thoughtful, trustworthy, and ethical. In addition, the belief that the servant leader who meets the needs of the follower’s cares about an active and quality relationship is also strengthened (Liden et al., 2008: 163). Creating a culture of trust is very important for servant leadership (Chon & Zoltan, 2019: 18).

Spiritual Leadership: Spiritual leadership is inspired by change-oriented leadership styles and is the behaviors, attitudes and values that support people to turn to intrinsic motivation through the call and membership method (Fry, 2003). Thus, the spiritual leader is defined as being able to internally motivate herself and her/his followers within the framework of certain values, attitudes and behaviors so that the followers have a sense of spiritual survival through calling and membership (Fry et al., 2005: 836). Spiritual leadership reveals the meaningfulness of work through intrinsic motivation, it also increases the level of commitment of employees to their job and the organization (Fry et al., 2017). The aim here (Fry, 2003: 695) creating a vision in which organizational members experience a sense of calling that their lives are meaningful and that they make a difference; to create a social/organizational culture that is based on caring, appreciative, and altruistic love and thus, creating a sense of membership and ensuring that one feels understood and appreciated. The spiritual leader is sensitive to the needs of her/his followers to become more organized and productive. They also develop the ability of followers to understand and solve problems they encounter through intellectual discourse (Fry et al., 2017). In this process, thanks to the hopes and beliefs of spiritual leaders, followers do not avoid facing challenges in their work (Jain, 2022). Organizations try to meet the spiritual needs of employees by aiming to satisfy their longing for meaning and purpose in their professional endeavors (Aboobaker & Zakkariya, 2023: 541), at this point, the spiritual leadership style for organizations helps employees discover the meaning of their work and gain spiritual well-being (Ali et al., 2022: 3914). Spiritual leadership emphasizes both rational and emotional components of leadership to motivate employees (Chen & Li, 2013) and takes care to create a value congruence between the organization's goals and the needs of followers (Fry et al., 2017). Additionally, spiritual leadership is being able to acknowledge and appreciate the contributions and supports of others (Fry et al., 2005) and it is also effective in facilitating social participation and developing a common vision through bottom-up interactive communication through appreciation (Wang et al., 2019).

Cross-Cultural Leadership: Although leadership is an important way to coordinate across the world, societal culture shapes leadership processes and effects (Den Hartog & De Hoogh, 2024). As connectivity increases, diversity within the organization also increases. Therefore, it is important for organizational leaders to be intercultural leaders. In the era of globalization where organizations operate in diverse cultural contexts, effective leadership across borders has become integral to success (Wadhera, 2024). With the acceleration and deepening of globalization, regional borders have become increasingly accessible. This raises the issue of how intercultural leaders can benefit from the positive aspects of their intercultural followers (Lyu, 2021: 1). Intercultural leadership is of great importance in transnational organizations. In fact, intercultural leadership is the person who can both adapt to the changes brought by globalization and adapt to global cultural fluctuations in terms of cultural change and evolution (Lyu, 2021: 2). In order to establish global collaborations, the leader must be culturally competent. Respecting cultural differences will make it easier for leaders to build trust, reduce misunderstandings, and improve communication. In intercultural leadership, communication and decision-making are adapted to the cultures of international stakeholders (Gao, 2024: 2).

Complexity Leadership: Complexity is related to the large number of interconnections. When objects interact with each other, they change each other in unexpected and irreversible ways. Here, the concepts of complex and complexity[2] emerge (Uhl-Bien & Arena, 2017: 9-10). The word complex comes from the Latin root plectere, meaning "to weave" or "to knit". In complex systems, many simple parts are irreducibly intertwined and the complexity domain itself is an interlocking of many different domains (Mitchell, 2022: 18). Many systems important to humans exhibit complexity. Some of these are markets, organized into groups that contribute to investment funds with their various buyers and sellers; economies, with their hierarchies of workers, departments, firms, and industries; single-celled organisms composed of proteins, membranes, organelles, cells, and organs; and the internet, with its users, stations, servers, and sites (Holland, 2020: 11-12). If the relationships in a system are not fixed but variable and therefore the system components cannot be fully explained by analysis, the system is complex. This complexity gives rise to new properties (self-organization), which are often called emergent properties (Uhl-Bien & Marion, 2009: 632). Complexity science has enabled the development of new leadership perspectives, leadership, called complexity leadership, is so complex that it cannot be defined as the action of only one individual or individuals and it is also described as a complex interaction of many forces interacting with each other (Uhl-Bien et al., 2007: 314). It means making room for change and innovation and taking advantage of the results of the natural interactions that occur in organizations (Schophuizen et al., 2023: 472), complexity leadership is seen as an alternative approach for organizations operating in a variable, unpredictable, competitive, chaotic environment based on information technology to survive

[2] Holland (2020: 13) notes that complexity, like life and consciousness, lacks a rigorous definition.

(Baltacı & Balcı, 2017: 32). It is important for complexity leaders to be able to create conditions in which they can respond quickly and effectively to unexpected circumstances (Marion, 2008: 10). Complexity leadership aims to exploit the dynamic capabilities of complex adaptive systems (CAS). Complex leadership focuses on identifying strategies that encourage creativity, learning, and adaptability.

Coaching Leadership: Coaching is facilitating growth and change by activating the skills that the individual has (DiGirolamo & Tkach, 2019: 197). In the most common terms, coaching enables an individual to get from where s/he is to where s/he wants to be. The leader-follower relationship, like the coach-client relationship, is based on trust. Zhang et al. (2024: 113) coaching leadership takes care to establish a supportive relationship with followers. Coaching leadership is not like mentoring and tutoring. Coaching leadership aims to increase short-term performance with instant feedback and helps followers see the obstacles to their performance, overcome those obstacles and set specific goals (Liu & Batt, 2010: 270-271). Dello Russo et al. (2017: 772) coaching leaders are people who help employees maximize their potential and abilities by paying attention to their needs and establishing an effective alliance. Coaching leaders focus on the motivation of their followers, their relationships with people, and acting within ethical principles.

2.2. Entrepreneurial Orientation

It is accepted that entrepreneurship is a fundamental characteristic of high-performance firms (narr. Lumpkin & Dess, 1996: 135) it is stated that opportunities and entrepreneurial behaviors are important in entrepreneurship (Kusa vd., 2021). An entrepreneurial firm is one that is competitive, engaging in product-market innovation, taking on risky ventures, and being the first to introduce "proactive" innovations. In contrast, a non-entrepreneurial firm is one that innovates little, is extremely risk averse, and imitates competitors' actions rather than taking the lead (Miller, 1983: 771). As can be understood from the strategic decisions and management philosophies of the organizations, it is emphasized that the top managers of the entrepreneurial organizations have an entrepreneurial top management style (Covin & Slevin, 1989: 77). In addition, the entrepreneurial capacity of an organization is also related to the entrepreneurial orientations and behaviors of its employees (Covin et al., 2020).

While entrepreneurial orientation represents how an organization organizes to discover and exploit opportunities (Wiklund & Shepherd, 2003: 1310), it is also expressed as a strategy creation process that characterizes the entrepreneurship of an organization (Shan et al., 2016: 684). The place and importance of technological knowledge in the process of discovering and using opportunities (Wiklund & Shepherd, 2003: 1309) should not be forgotten. Hughes et al. (2015: 119) state that entrepreneurial orientation can be an important antidote to the pressures arising from the rapidly changing environment and the threats brought about by the decrease in flexibility rate as the age/size of organizations increases. Covin et al. (2020: 2) therefore, entrepreneurial orientation can provide flexibility and adaptability to large organizations. Individual entrepreneurship orientation is based on the idea that entrepreneurs can be an economic resource when they can implement their entrepreneurial skills, which include recognizing and searching for applicable business opportunities and successfully using these business opportunities (Santos et al., 2020: 193). Miller (1983) addressed entrepreneurial orientation as innovativeness, proactiveness and risk taking and stated that these are the most studied behaviors or characteristics in the field of entrepreneurship (Wiklund, 1999: 38).

Innovation: With the shortening of product life cycles, intensification of competition, and rapidly changing business environments, innovation and the speed of innovation have a significant impact on organizational results (Shan et al., 2016: 683) and it affects the survival, sustainability, competitiveness and growth of organizations (Alrowwad et al., 2020: 201). Lumpkin and Dess (1996) innovation; creativity and experimentation in delivering new products or services; innovation in the development of new processes is expressed as technological leadership and willingness to support research and development. Covin et al. (2020: 3) defining innovation as the employee's ability to adapt to the job and search for innovations related to the job, Moustaghfir et al. (2020: 270) define innovation as the ability to be transferred to an organizational context through a business tendency to encourage the creation of new products, services and processes. Alrowwad et al. (2020: 201) The organizational structure can provide the internal configuration necessary for innovation to take place, thus providing the necessary inputs for innovation and creating an advantage for the organization. In order for innovation to take place and become an advantage, Covin et al. (2020: 3) the employee must exhibit entrepreneurial behavior in order to increase organizational performance and Ritala et al. (2021: 3) the employee must have innovation competence at the employee level. In addition, organizations with an entrepreneurial spirit will gain the power to challenge competitors with their innovative features (Ercan & Yıldıran, 2021: 143).

Proactivity: Venkatraman (1989: 949) defines proactivity as the search for new opportunities that may or may not be related to the current field of activity, the introduction of new products, or the strategic elimination of operations that are in decline in their life cycle. Lumpkin and Dess (1996; 2001: 431) define proactivity as a future-oriented perspective that includes offering new products and services in order to compete, creating changes, and shaping the environment by predicting future demands. Finally, Covin et al. (2020: 3) define proactivity as being able to anticipate and respond to new value creation opportunities. If employees in the organization are entrepreneurship-oriented, it can be said that they can act proactively and direct both their time and resources to entrepreneurial opportunities to implement change.

Risk Taking: Lumpkin and Dess (1996) define risk taking as the tendency to take bold actions, such as entering new markets and investing resources in ventures with uncertain outcomes. Risk taking is measured through managers' preferences for bold and cautious behavior so that organizations can achieve their goals (Kusa, 2021: 235). A calculated risk can be taken to reduce the probability of failure. However, in an environment where risks can be inevitable at any time, having a positive attitude towards taking risks has become a necessity (Krauss et al., 2005: 321). Sometimes time may be limited to fully calculate each risk and being the earliest to take action rather than calculating it may provide an advantage to organizations. Due to the intense competition in globalizing markets, organizations in every sector and location need to focus on increasing their entrepreneurial orientation both to maintain their market shares and to increase their existing market shares (Guzmán et al., 2020: 1078).

3. Method

PES is an MCDM method based on three indicators: security level, mean level, and optimism level. Its stages are as follows when criteria weights are equal (Güçlü & Göktaş, 2023; Göktaş & Güçlü, 2024b).

Stage 1: Decision matrix (A) is formed.

Stage 2: Normalized decision matrix (B) is formed using Equation 1. a_{ij} corresponds to the original value of the i^{th} alternative for the j^{th} criterion, whereas b_{ij} corresponds to the normalized value of the i^{th} alternative for the j^{th} criterion.

$$b_{ij} = \begin{cases} \dfrac{a_{ij} - \min\limits_i a_{ij}}{\max\limits_i a_{ij} - \min\limits_i a_{ij}}, \text{if utility criterion} \\[2ex] \dfrac{\max\limits_i a_{ij} - a_{ij}}{\max\limits_i a_{ij} - \min\limits_i a_{ij}}, \text{if cost criterion} \end{cases} \qquad (1)$$

Stage 3: Security level (s_i) is attained as the minimum of the i^{th} row of B. Mean level (m_i) is attained as the average of the i^{th} row of B. Optimism level (o_i) is attained as the maximum of the i^{th} row of B.

Stage 4: Performance value (p_i) is calculated using (2). The sum of them equals 1.

$$p_i = \frac{1}{\sum_i \frac{(s_i + 2m_i + o_i)}{(1 - s_i)^2}} \frac{(s_i + 2m_i + o_i)}{(1 - s_i)^2} \tag{2}$$

Stage 5: Alternatives are ranked in descending order using the performance values.

3. Results

This section uses PES to compare the leadership types in terms of entrepreneurship. Seven leadership types are alternatives, and three dimensions of entrepreneurship are criteria. Alternatives and criteria are examined in Section 2. Three decision matrices are separately obtained using three AI chatbots: ChatGPT4, Gemini, and Copilot. AI chatbots rank the alternatives for each criterion from 1 to 7. 1 is used for the first rank, and 7 is used for the last rank. A similar procedure is used by Göktaş and Güçlü (2024a). The stages of PES for this MCDM problem are as follows.

Stage 1: The decision matrices are separately formed as in Table 1, Table 2, and Table 3.

Table 1: The decision matrix formed by ChatGPT4.

	Innovation	Proactivity	Risk-taking
Transformational	1	1	2
Transactional	7	5	7
Servant	4	3	4
Spiritual	5	6	6
Cross-Cultural	3	4	3
Complexity	2	2	1
Coaching	6	7	5

Table 2: The decision matrix formed by Gemini.

	Innovation	Proactivity	Risk-taking
Transformational	1	2	3
Transactional	7	7	5
Servant	4	5	7
Spiritual	2	3	4
Cross-Cultural	6	4	2
Complexity	3	1	1
Coaching	5	6	6

Table 3: The decision matrix formed by Copilot.

	Innovation	Proactivity	Risk-taking
Transformational	1	2	3
Transactional	4	5	6
Servant	7	6	5
Spiritual	2	1	4
Cross-Cultural	5	4	2
Complexity	3	3	1
Coaching	6	7	7

Stage 2: The normalized decision matrices are separately formed using (1) as in Table 4, Table 5, and Table 6.

Table 4: The normalized decision matrix according to ChatGPT4.

	Innovation	Proactivity	Risk-taking
Transformational	1.000	1.000	0.833
Transactional	0.000	0.333	0.000
Servant	0.500	0.667	0.500
Spiritual	0.333	0.167	0.167
Cross-Cultural	0.667	0.500	0.667
Complexity	0.833	0.833	1.000
Coaching	0.167	0.000	0.333

Table 5: The normalized decision matrix according to Gemini.

	Innovation	Proactivity	Risk-taking
Transformational	1.000	0.833	0.667
Transactional	0.000	0.000	0.333
Servant	0.500	0.333	0.000
Spiritual	0.833	0.667	0.500
Cross-Cultural	0.167	0.500	0.833
Complexity	0.667	1.000	1.000
Coaching	0.333	0.167	0.167

Table 6: The normalized decision matrix according to Copilot.

	Innovation	Proactivity	Risk-taking
Transformational	1.000	0.833	0.667
Transactional	0.500	0.333	0.167
Servant	0.000	0.167	0.333
Spiritual	0.833	1.000	0.500
Cross-Cultural	0.333	0.500	0.833
Complexity	0.667	0.667	1.000
Coaching	0.167	0.000	0.000

Stage 3: Security levels, mean levels, and optimism levels are separately calculated for each normalized decision matrix as in Table 7, Table 8, and Table 9.

Table 7: The indicators according to ChatGPT4.

	Security Level	Mean Level	Optimism L.
Transformational	0.833	0.944	1.000
Transactional	0.000	0.111	0.333
Servant	0.500	0.556	0.667
Spiritual	0.167	0.222	0.333
Cross-Cultural	0.500	0.611	0.667
Complexity	0.833	0.889	1.000
Coaching	0.000	0.167	0.333

Table 8: The indicators according to Gemini.

	Security L.	Mean L.	Optimism L.
Transformational	0.667	0.833	1.000
Transactional	0.000	0.111	0.333
Servant	0.000	0.278	0.500
Spiritual	0.500	0.667	0.833
Cross-Cultural	0.167	0.500	0.833
Complexity	0.667	0.889	1.000
Coaching	0.167	0.222	0.333

Table 9: The indicators according to Copilot.

	Security L.	Mean L.	Optimism L.
Transformational	0.667	0.833	1.000
Transactional	0.167	0.333	0.500
Servant	0.000	0.167	0.333
Spiritual	0.500	0.778	1.000
Cross-Cultural	0.333	0.556	0.833
Complexity	0.667	0.778	1.000
Coaching	0.000	0.056	0.167

Stage 4: Performance values are separately calculated using (2) as in Table 10.

Table 10: Performance values according to AI chatbots.

	ChatGPT4	Gemini	Copilot
Transformational	0.4698	0.3870	0.3787
Transactional	0.0019	0.0072	0.0242
Servant	0.0319	0.0136	0.0084
Spiritual	0.0048	0.1376	0.1543
Cross-Cultural	0.0335	0.0372	0.0647
Complexity	0.4557	0.3999	0.3661
Coaching	0.0023	0.0175	0.0035

Stage 5: Alternatives are separately ranked using the performance values. For example, transformational leadership ranks first according to ChatGPT4 and Copilot, whereas complexity leadership ranks first according to Gemini.

Table 11: The ranks of the alternatives according to AI chatbots.

	ChatGPT4	Gemini	Copilot	Borda Count
Transformational	1	2	1	1
Transactional	7	7	5	7
Servant	4	6	6	5
Spiritual	5	3	3	3.5
Cross-Cultural	3	4	4	3.5
Complexity	2	1	2	2
Coaching	6	5	7	6

The linear correlation coefficients between the different rankings are between 0.750 and 0.821. That is, AI chatbots present different but highly correlated rankings (Hair et al., 2007).

The Borda count method combines different rankings using the ranking sums (Aydın & Gümüş, 2022). It presents the general rankings as 1) transformational leadership, 2) complexity leadership, 3) and 4) spiritual (cross-cultural) leadership, 5) servant leadership, 6) coaching leadership, and 7) transactional leadership. It is noticed that spiritual and cross-cultural leaderships have the same rank. It should be noted that these rankings depend on three issues: the expert opinion, the MCDM method, and the criteria weights.

5. Discussion and Conclusions

The selection of leadership styles is a complex and difficult process for organizations. In this process, harmony between organization and leader and leader and follower relationship are important. It should not be forgotten that entrepreneurship is important in the world economy, the development of countries and the growth of organizations. As a result of the analysis, the general rankings are as follows.

The leadership style that is most closely related to entrepreneurial orientation is transformational leadership. Transformational leadership is defined as the effects and results a leader creates on her/his followers. Judge and Piccola (2004) state that transformational leadership provides a purpose that focuses on higher-level internal needs beyond short-term goals. When transformational leadership and entrepreneurial orientation are examined in current studies in the literature, transformational leadership; Paudel (2020) has a strong entrepreneurial orientation, Haase and Franco (2020) is important in promoting collective entrepreneurship, Malik et al. (2020) its positive relationship with work efficiency, Iqbal et al. (2021) its importance for innovation, Gerards et al. (2021) its promotion of entrepreneurial behavior, Sari et al. (2021) its relationship with social entrepreneurship, Xu and Jin (2022) positive effects on their relationships with entrepreneurs, Schiuma et al. (2022) promotes organizational innovation and digital transformation entrepreneurship, Yas et al. (2023) is the best strategic leadership, Deng et al. (2023) emphasizes that it has positive effects on individual, team and organizational outcomes, Luu (2023) that it is important for strategic entrepreneurship, Klein (2023) its effect on entrepreneurial behavior, and Ravet-Brown et al. (2024) measures leadership in entrepreneurship.

Second in line is complexity leadership. Considering that the conditions and environment are changing rapidly and that these changes are interdependent and interconnected, leadership approaches that were useful in the past may no longer be beneficial in the period called the new world order. A complexity leader must be able to recognize and adapt to complexity. So, complexity leadership is about being able to understand complex patterns and be productive in the unknown future. The leader creates environments where followers can organize themselves rather than control their individual outcomes. Thus, the leader can ensure the development of dynamic interactions within the organization. Considering this, it can be said that it is not surprising that complexity leadership comes second. When comlexity leadership and entrepreneurship orientation are examined in current studies in the literature, it is seen that complexity leadership, Khan et al. (2021) stated that it supports its relationship with entrepreneurial orientation, and Schophuizen et al. (2023) that it supports innovation management.

It has been found that spiritual and cross-cultural leadership have the same ranking. A spiritual leader is someone who is intrinsically motivated. A motivated leader will work to ensure that her/his followers are also intrinsically motivated. S/he will exhibit values, attitudes and behaviors in this direction. In the leader and follower interaction, there will be a distinct feeling of being understood and appreciated. When spiritual leadership and entrepreneurship orientation are examined in current studies in the literature, it is seen that spiritual leadership; Widyanti and Basuki (2021) emphasizes that it has a positive and significant relationship with innovative work behavior, Usman et al. (2021) encourages internal entrepreneurial behaviors of followers, Khaddam et al. (2023) has an effect on creative behavior, and Arshad and Saleem (2024) increases entrepreneurial behaviors within the organization. Globalization has created business models on a global scale, and these models have created the need for multicultural leadership. A leader who is successful in cross-cultural relations will increase the satisfaction between the parties. In order for cross-cultural leadership to be effective, inclusiveness, the ability to manage cultural differences and build cross-cultural trust are important. When cross-cultural leadership and entrepreneurial orientation are examined in current studies in the literature, it is seen that cross-cultural leadership; Lee and Kelly (2019) partially explain the differences in cross-national social entrepreneurship, while Muralidharan and Pathak (2019) state its impact on the likelihood of individuals becoming social entrepreneurs.

Fifth is servant leadership. The goal of a servant leader is to be of service to others and to ensure that their needs are met. When servant leadership and entrepreneurship orientation are examined in current studies in the literature, it is seen that servant leadership; Mallen Broch et al. (2020) emphasize that it has a positive relationship with organizational innovation, Alikhani and Shahriari (2022) found that entrepreneurial orientation positively and significantly mediates the relationship between servant leadership and competitiveness, Suhartanti and Prasetyanto (2022) emphasize that there is a positive and significant effect between innovation self-efficacy and employee productivity.

Coaching leadership comes in sixth place. Coaching is a process of development and change in which the coach and the client come together, enabling the client to get to know themselves, discover what they can do and reveal their potential. In this way, the way to achieve the goals will be provided (Eren & Yıldıran, 2023: 25). Coaching skills have become commonplace in organizations. Leaders are expected to have coaching skills. Leaders who have coaching skills are responsible for personal transformation to increase the well-being and performance of their employees. When current studies in the literature examine coaching leadership and entrepreneurship orientation, it is seen that coaching leadership; Kim and Oh (2021) state that it facilitates innovative behavior among employees, while Hwang et al. (2023) state that it has a positive effect on creative performance.

Finally, in the last place is transactional leadership. In transactional leadership, optimum use of resources is aimed. Therefore, in order to get her followers to do something, the leader gives them something they want in return. It can be expressed as an exchange that progresses in line with the personal interests of leaders and followers. As seen in the literature, it can also be in the form of conditional rewards or exceptional management. When transactional leadership and entrepreneurial orientation are examined in current studies in the literature, transactional leadership; Nungky Viana et al. (2020) stated that it has a direct positive and insignificant effect on the innovation of SMEs, Thahira et al. (2020) stated that it has an effect on innovation, Udin et al. (2022) stated that it has no effect on innovative behavior, Klein (2023) stated that it has a positive effect on intrapreneurial behavior.

One of the most important organizational challenges faced by leaders in the 21st century is the necessity of constantly developing a new business model without neglecting the economic performance of businesses (Sapta vd., 2021: 3). It becomes important which leadership models both organizations and leaders will choose and implement. It is not surprising that transformational leadership comes first when its attitude towards followers and ways of motivating them are considered. Transformational leaders encourage innovation in line with their followers' goals and ideas. With globalization and technology, there is a transformation from production to knowledge in both the economy and organizations. This transformation means more information and connections. It is known that the inspiration for complexity leadership is complexity science. In this regard, managing complex adaptive system dynamics is important. As a result of the study, it is seen that complexity leadership is in second place. The reason why spiritual and intercultural leadership appear in the same ranking can be stated as follows. The spread of transnational collaborations and therefore the increasing importance of leaders developing themselves culturally within the framework of cultural values and leading in this direction. It shows that although there is a globalizing business life, individuals' search for spirituality or their efforts to make sense of work/business life also progress in parallel. In addition, it is an important factor that spiritual leadership is based on ethical and moral values. It can be stated that servant leaders are ranked fifth because they help their followers develop so that they can achieve career success and thus have an impact on their entrepreneurial orientation. Coaching leadership also identifies the needs and talents of followers and supports their development, guiding them towards higher goals. However, because spiritual awareness is higher in servant leadership, it came in fifth place. The content of leader-follower relationships affects entrepreneurial orientation as a whole (individual-team-organization). The leadership that comes last in the ranking is transactional leadership. Transactional leadership rewards those with high productivity among followers. It punishes those who are low in productivity or do not complete their tasks. It tries to manage and encourage the performance of the followers through reward and punishment method. For this reason, it can be stated within the framework of the findings that there is not a very effective leadership in terms of entrepreneurial orientations.

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CHAPTER IV

Application of Fuzzy Linear Programming to the Turkish Cement Sector

Rehile ASKERBEYLİ[1]
Levent ÜNALAN[2]

1. Introduction

1.1. Definition And History of Fuzzy Logic

In real life, some situations are too complex to be expressed clearly. These situations may not be mathematically expressed or controlled (Erdoğan, 2003). Fuzziness is the term given to the uncertainty or lack of precision in the expression of a concept or purpose (Gülcan, 2012; Askerzade, 2010; Ziasabounchi, 2014). The model used to mathematically express these uncertain situations, which fall outside classical logic, is called "Fuzzy Logic."

When considering the historical development of fuzzy logic, it should be examined in three separate periods: the ancient period,

[1] Prof. Dr. Rehile ASKERBEYLI, Karabuk University, Faculty of Business, Department of Business Administration, Karabük/Turkey, Orcid: 0000-0001-6417-0939, raskerbeyli@karabuk.edu.tr
[2] Levent ÜNALAN, Karabuk University, Faculty of Business, Department of Business Administration, Karabük/Turkey.

the modern period, and the period of fuzzy logic. The philosophical foundations of this logic were laid by thinkers such as Master Mo, Plato, Socrates, Aristotle, Parmenides, and Zeno of Elea. Master Mo, considered the founder of the Confucian and Mohist schools, worked on establishing rules for correct reasoning and deriving correct conclusions. These studies are thought to date back to the 4th century BCE. Socrates first addressed the concept and laid the foundations of the system of inductive reasoning, while Plato argued that the concept of being cannot be created with inductive techniques and stated that reality lies in the world of ideas beyond the sensory and experiential universe. Some of the key figures who made significant contributions to the foundations of logic in the modern period include Albertus Magnus (1193-1280), Francis Bacon (1561-1626), and Immanuel Kant (1724-1804).

The concept of fuzziness was first used by the American philosopher Black. Subsequently, due to the work of scientists, the concept of Fuzzy Logic was discussed in the second half of the 20th century and defined with fuzzy sets.

The theory was first defined in 1965 by the Azerbaijani scientist Lotfi A. Zadeh in a published paper. In this paper, Zadeh noted that a large part of the human mind is fuzzy and argued that fuzzy logic, when controlled using the fuzzy logic method, performs much better than classical logic (Zadeh, 1965). In 1972, Sugeno introduced a new perspective to fuzzy logic by presenting the concepts of fuzzy integrals and fuzzy measures (Sugeno, 1972). Mamdani applied fuzzy logic in the control phase of a steam engine in 1974 (Mamdani, 1974). Zimmermann laid the foundations of fuzzy optimization with his work (Zimmermann, 1978). In 1988, during the crisis known as "Black Monday" on the Tokyo Stock Exchange, a system developed by Yamaichi Securities, based on fuzzy logic, provided an early warning exactly 18 days in advance. Following these successful results, fuzzy logic gained popularity, and companies with well-known brands such as IBM, Matsushita, Omron, Thomson, SGS, and Toshiba established their Laboratory for Interchange Fuzzy Engineering (LIFE) in 1989, using fuzzy logic in significant technological advancements.

1.2. Fundamental Principles of Fuzzy Logic

The concepts used to express or explain something generally have a fuzzy structure. These concepts can be verbal or numerical expressions and contain fuzziness. For example, when describing something, we can use terms like a little, some, a lot, and too much. These expressions, which do not indicate certainty, are examples of situations that are not definite and contain uncertainty in the human mind. For instance, the terms very thin, thin, overweight, and very overweight used to describe a person's weight are fuzzy and indefinite expressions, resulting from fuzzy logic. In fuzzy logic, it is possible to define sharp expressions as well as intermediate values. Fuzzy logic can be examined in a narrow and broad sense. In the narrow sense, it represents the generalized state of classical binary logic. In the broad sense, it encompasses all theories and technologies that use fuzzy sets.

According to Zadeh, the principles of fuzzy logic are:

- Instead of precise values, approximate values are used in fuzzy logic.

- In fuzzy logic, all values are expressed with a membership degree between [0-1].
- All logical expressions can be converted into fuzzy expressions.
- Information is described with verbal expressions such as very little, little, a lot, and very much.
- Fuzzy logic can be used as a suitable method when the systems modeled mathematically are very complex and difficult.

1.3. Fuzzy Set

Sets whose elements can have different membership degrees between [0-1] are called Fuzzy Sets. The difference between the theory of fuzzy sets and classical set theory is that it allows partial membership for elements in the set. Fuzzy sets encompass classical sets, meaning that a classical set is a subset of a fuzzy set. Binary logic theory states that an element either belongs to a set or does not; if it belongs, it takes the value of 1, and if not, it takes the value of 0. In a fuzzy set, there is no such certainty. In this case, the principle of the excluded middle and the principle of non-contradiction in classical logic do not apply in fuzzy logic. Fuzzy sets have properties such as union, intersection, and complement. Membership values indicate the degree to which an object belongs to a fuzzy set. In a fuzzy set, the membership of an object is expressed by a membership function, not a characteristic function.

The membership function in a fuzzy set is the function that determines the membership degree of the elements belonging to this set. The membership function is what allows the fuzzy set to be graphed. On the graph, the X-axis represents the input universe, and the Y-axis represents the membership degree in the range [0,1]. In the membership function, often denoted by μ_A, the value $\mu_A(x)$ represents the membership degree of the element x. (Omar et al., 2015).

In Classical Logic, the Membership Function is:

$$\mu_A(x) = \begin{cases} 1; & x \in A \\ 0; & x \notin A \end{cases}$$

In Fuzzy Logic, the Membership Function is:

$$\mu_A(x) = E\ [0,1]$$

Parts of the Membership Function in Fuzzy Logic

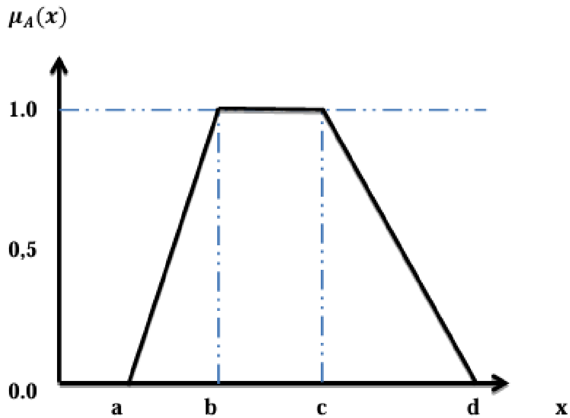
- Core: In this region, the membership function takes the value of 1. It includes elements with the full membership degree of the set (Zimmermann, 1991). $\mu_{A(x)=}1$
- Support: This is the part where the membership function is greater than 0. $\mu_A(x) > 0$
- Boundary: This is the part where elements have a partial membership degree, not equal to 0 or 1 (Sivanandam et al., 2007). $0 < \mu_A(x) < 1$
- Height: The highest membership degree of the set is its height (Zimmermann, 1991). $\max[\mu_A(x)]$

Types of membership functions commonly used in practice and in the literature (Şen, 2004):

- Trapezoidal Membership Function
- Triangular Membership Function
- Bell Curve Membership Function

- Gaussian Membership Function
- Sigmoidal Membership Function

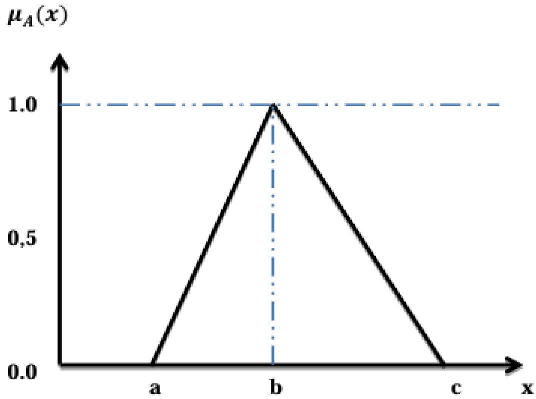
Figure 1: The parts of a trapezoidal membership function.



Mathematical expression of the trapezoidal membership function:

$$\mu_{\underset{\sim}{A}}(x) = \begin{cases} 0, & x \le a \\ \dfrac{x-a}{b-a}, & a \le x \le b \\ 1, & b \le x \le c \\ \dfrac{d-x}{d-c}, & c \le x \le d \\ 0, & d \le x \end{cases}$$

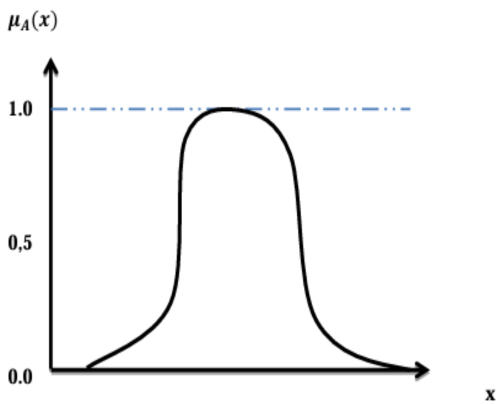
Figure 2: The parts of a triangular membership function.



Mathematical expression of the triangular membership function:

$$\underset{\sim}{\mu_A}(x) = \begin{cases} 0, & x \le a \\ \dfrac{x-a}{b-a}, & a \le x \le b \\ \dfrac{c-x}{c-b}, & b \le x \le c \\ 0, & c \le x \end{cases}$$

Figure 3: The parts of a Gaussian (Bell Curve) membership function.



This type of membership function is expressed by the parameters m and σ. General formulation,

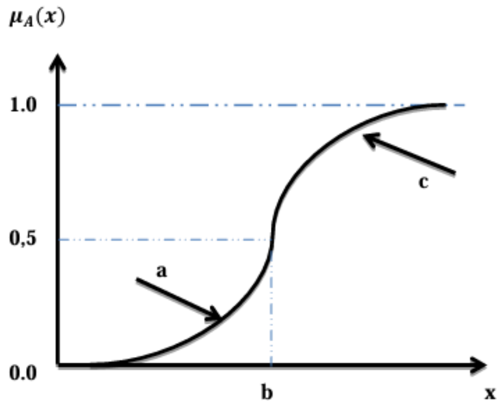
$$\mu_{\underset{\sim}{A}}(x) = \exp\left\{\frac{-(x-m)^2}{2\sigma^2}\right\}$$

Where m is the function centre and σ is the width. Consequently, when σ changes, the form of the function also changes.

The sigmoidal membership function type has two parameters such as a and c. Its mathematical representation is

$$\mu_{\underset{\sim}{A}}(x) = \frac{1}{1+ae^{-(x-c)}}$$

Figure 4: The parts of a sigmoidal membership function.



This type of membership function exhibits openness to the right or left depending on whether the parameter 'a' is positive or negative, and is frequently used to represent fuzzy terms such as “very large”, “very small”, or “fairly”. In sigmoidal membership functions, there is a breakpoint between belonging and not belonging, and this value is the 'b' parameter.

1.4. Fuzzy Numbers

A fuzzy set Ã defined in the real number set R can become a fuzzy number when certain conditions are met (Özkan, 2003). These conditions are:

- If it has a normal and convex membership function,
- If the support set is limited ($\alpha \in (0,1]$),
- Each α-cut set must be defined within a closed interval on the real number line.

A fuzzy number is a special case of fuzzy sets, and thus, it is defined by a membership function as in fuzzy sets. Consequently, in the literature, the type of fuzzy number corresponds to the type of membership function (Baykal & Beyan, 2004).

In practical applications, triangular fuzzy numbers are the most preferred for arithmetic operations. Given two positive fuzzy numbers A and B, where Ã = (a1, a2, a3) and B̃ = (b1, b2, b3), the arithmetic operations are as shown in Table 1.

Table 1: Arithmetic operations with triangular fuzzy numbers.

Ã+B̃	(a1, a2, a3)+ (b1, b2, b3)= (a1 +b1, a2+ b2,a3+ b3)
Ã-B̃	(a1, a2, a3)- (b1, b2, b3)= (a1 -b1, a2- b2,a3- b3)
Ã×B̃	(a1, a2, a3) × (b1, b2, b3)= (a1×b1, a2× b2,a3× b3)
Ã÷B̃	(a1, a2, a3): (b1, b2, b3)= (a1/b3, a2/b2,a3/ b1)
$(\tilde{A})^{-1}$	$(a1, a2, a3)^{-1}$=(1/a1,1/a2,1/a3)

1.5 Advantages and Disadvantages of Fuzzy Logic

Compared to classical logic, fuzzy logic has several advantages):

- The control operations are expressed depending on linguistic variables,
- It yields successful results in dynamic and nonlinear systems that are difficult or impossible to model mathematically because fuzzy logic does not require a mathematical model,
- It can operate with uncertain and incomplete data,
- It can quickly reach results using only a few membership functions in applications involving extensive data,
- It is a more suitable approach than classical numerical methods for conveying human thinking processes, making it superior to classical numerical approaches.
- There is no universal method for analyzing observability, controllability, and stability of systems, necessitating expert opinions and experience to define fuzzy inference rules,
- There is no absolute method for determining membership functions, often requiring a trial-and-error approach that can be time-consuming,
- Membership functions are specific to the created system, making it difficult to adapt them to other systems.

1.6. Decision-Making in a Fuzzy Environment

Decision-making involves selecting the most appropriate action from among several possible actions to achieve a goal. The decision process may favour one or more strategies to achieve the goal. Fuzzy decision-making involves elements such as the decision maker, goal, decision criteria, options, events, and outcomes. The goal and decision criteria are considered fuzzy objectives. Parameters and/or right-hand side constants that specialise events can be made fuzzy. Some tolerances can be added to constraints in $\geq$, $=$, $\leq$ relationships. This component is considered to be a constraint.

Fuzzy decisions, a subset of fuzzy goals and constraints, exhibit the satisfaction degree of fuzzy constraints and the performance of encountered fuzzy targets. According to the rule expressed as achieving $\tilde{G}$ target and satisfying $\tilde{C}$ constraint, the fuzzy decision set is mathematically determined as $\tilde{D} = \tilde{G} \cap \tilde{C}$ (Özkan, 2003). For optimal decisions, the fuzzy decision with the highest membership degree is chosen.

1.7. Fuzzy Linear Programming

Fuzzy linear programming, a combination of fuzzy logic and linear programming, extends classical linear programming. While classical linear programming aims to maximise or minimise (optimise) the objective function based on variables and constraints, real life conditions often render constraints and objective functions indeterminate. Fuzzy linear programming, introduced by Zimmermann in 1978, aims to incorporate uncertainties in decision making into the model (Hansen, 1996). Unlike classical linear programming, fuzzy linear programming uses the fuzzy symbol '~' in parameters or constraints. It can be applied to solve problems in human resource management, production, banking, finance, and agricultural economics.

The most common representation of fuzzy linear programming problems is:

Objective Function - $Max\ Z = \sum_{j=1}^{n} \widetilde{cj}.\widetilde{xj}$

Constraints: Non-negativity Constraint:

Fuzzy linear programming is a technique used to optimise decision-making processes in situations involving uncertainty. The Werners and Zimmermann approaches offer two different methods in this area. These methods adopt different ways to deal with fuzzy data and objectives.

Fuzzy linear programming method proposed by Zimmermann aims to find solutions to problems with fuzzy objective functions and fuzzy constraints. In this approach, fuzzy sets are defined for each constraint and objective function and the problem is solved using these sets.

Zimmermann was the first to use the fuzzy linear programming model. According to Zimmermann, a fuzzy constraint obtained from the decision maker expresses a fuzzy objective function with a fuzzy access level. Consequently, the resulting model is symmetric. Zimmermann also states that both the objective and the constraints are defined fuzzily by the decision maker, so when determining the fuzzy decision set, fuzzy objectives and constraints are evaluated indistinguishably. Zimmermann suggests that a desired level Z can be set for the objective function aimed by the decision maker, and constraints can be modeled individually as a fuzzy set (Zimmermann, 1991). The main steps of the Zimmermann approach are as follows:

Fuzzification of the objective function: The objective function is optimized based on a fuzzy goal. Typically, this goal represents achieving a satisfactory level for the objective function (e.g., "good" or "excellent").

Fuzzification of constraints: Constraints are relaxed within a certain tolerance range, allowing the solution process to adapt to uncertainty.

Linguistic variables: Constraints and objective functions are expressed linguistically (e.g., "high profit", "low cost").

Combined objective function: All fuzzy objectives and constraints are combined using a total satisfaction degree (membership function), which is then maximized.

Zimmermann's approach extends classical linear programming by incorporating fuzzy objective functions and constraints into the optimization process. The solution typically seeks to maximize the level of satisfaction according to fuzzy logic rules.

The general representation of the model proposed by Zimmermann is:

$$c^T . x \gtrsim b_0$$

$$(Ax)\ \boldsymbol{i} \lesssim b_{\boldsymbol{i}}\ \forall\ \boldsymbol{i}\ \text{for}$$

$$x \geq 0$$

The fuzzy inequalities $\tilde{\leq}$ and $\tilde{\geq}$ in the model indicate that the expression (Ax) is around or less than the value b and the expression

$c^T.x$ is around or more than the value b_0 (Özkan, 2003). The piecewise linear functions of the fuzzy objective function and constraints are respectively:

$$\mu_0(x) = \begin{cases} 1; & if\ c^T x > Z^1 \\ 1 - \dfrac{Z^1 - c^T x}{Z^1 - Z^0}; & if\ Z^0 \leq c^T x \leq Z^1 \\ 0; & if\ c^T x < Z^0 \end{cases}$$

$$\mu_{i1(x)} = \begin{cases} 1; & if\ (Ax)_i < b_i \\ 1 - \dfrac{(Ax)_i - b_i}{p_i}; & if\ b_i \leq (Ax)_i \leq b_i + p_i \\ 0; & if\ (Ax)_i > b_i + p_i \end{cases}$$

When λ is used as an additional variable in symmetric fuzzy linear programming models, the forms of the fuzzy objective and constraints change by altering their membership functions:

Max λ

Constraints

$c^T.x \geq b_0-(1-\lambda)p_0$

$(Ax)_i \leq b_i+(1-\lambda)p_i$

$x_i \geq 0$

Werners' approach takes a slightly different path than Zimmermann's. Instead of expressing the objective function and constraints as fuzzy numbers, Werners provides a more flexible preference structure for decision-makers. In this method, a hierarchy is created for the objective functions, and optimization is carried out based on this hierarchy.

The key features of the Werners approach include:

Multi-objective fuzzy programming: When there are multiple fuzzy objective functions, a hierarchy or priority order can be established among these objectives.

Fuzzy objectives and flexibility: The objective functions are expressed with flexibility, allowing decision-makers to choose between different solution options.

Levels of flexibility: A specific level of satisfaction or degree of flexibility is defined for each objective function, and a balance is struck between these levels.

Pareto optimality: Werners focuses on Pareto optimal solutions, aiming to find the best solution where improving one objective does not worsen another.

In 1987, Werners stated that because the constants on the right side are fuzzy, the objective function should be fuzzified. According to Werners, the right-side fuzzy linear programming model should first have its right side constants fuzzified, followed by the objective function. Given all this, Werners Model is symmetric. Orlovski's proposed fuzzy decision set is used to determine the membership function in Werners' objective function. Again, Orlovski suggests that for each α-cut set of the definition set consisting of fuzzy constraints, the optimal values of the objective function should be found, and this optimal value should be accepted as the fuzzy decision set in the solution space with equal membership degree. According to Werners’ fuzzy linear programming approach, the model is:

$$\text{Max } Z=c^T x$$

Constraints

$(Ax) \widetilde{\leq} b_i \quad i=1,2,\ldots..m$

$x_i \geq 0$

The optimal value will be found in the range Z_0 and Z_1, and the membership function written for the objective function in this range will be a continuously increasing linear function. If the membership function of the objective function and fuzzy constraints is shown:

$$\mu_0(x) = \begin{cases} 1; & if \;\; c^T x > Z^1 \\ 1 - \dfrac{Z^1 - c^T x}{Z^1 - Z^0}; & if \; Z^0 \leq c^T x \leq Z^1 \\ 0; & if \; c^T x < Z^0 \end{cases}$$

$$\mu_i(x) = \begin{cases} 1; & if \;\; (Ax)_i < b_i \\ 1 - \dfrac{(Ax)_i - b_i}{p_i}; & if \;\; b_i \leq (Ax)_i \leq b_i + p_i \\ 0; & if \; (Ax)_i > b_i + p_i \end{cases}$$

To determine the optimal decision, Werners' method is symmetric because the max (min) operator is used. In the fuzzy linear programming model, the satisfaction of the objective function and constraints is achieved together. To reach the optimal decision, the min operator proposed by Bellman and Zadeh is used, and the decision domain $\tilde{D}$ found with the membership function μ_D is obtained:

$\mu_D = \min(\mu_0, \mu_1 \ldots \mu_m)$

Werners model can easily be converted to a classical linear programming model, and the equality will be as follows if the maximum of the optimal solution is chosen as the decision in the equality μ_D:

Max λ

$\mu_0 \geq \lambda$

$\mu_1 \geq \lambda$

$\lambda \in [0,1]$, $\mu_0 \in [0,1]$, $\mu_i \in [0,1]$, $\forall$ i için $x \geq 0$

It has been previously mentioned that to achieve the optimal decision, the fuzzy objective function and fuzzy constraints must be satisfied together. If the min operator proposed by Zadeh and Bellman is used for this purpose:

$$\mu_{Decision} = \lambda = \min (\mu_{objective\ function} , \mu_{constraints})$$

$\mu_{Decision}$ defines an increasing membership function, which can be found by maximizing the membership degrees (λ) that satisfy the objective function and constraints at the same time as the classical linear programming model.

$\mu_{objectivefunction} \geq \lambda$

$\mu_{constraints} \geq \lambda$

$x \geq 0$, $\lambda \in [0,1]$

Chanas introduced a new perspective on fuzzy linear programming models with fuzzy objectives and constraints, suggesting that it is unrealistic to determine the decision maker's target level without obtaining any information. This view diverges from Zimmermann's, as Chanas accepted parametric programming as the basis for solving symmetric fuzzy linear programming problems.

In the Vergeday approach, the solution to the fuzzy linear programming model with fuzzy constraints is reached using the description theorem and parametric programming. To achieve a fuzzy solution in a fuzzy-constrained linear programming model, the fuzzy constraint must be divided into α-cut sets. The decision maker entirely determines which of the solutions calculated by parametric programming will be chosen as the fuzzy linear programming solution (Özkan, 2003).

Carlsson and Korhonen have also worked on fuzzy linear programming models with fuzzy parameters. According to their perspective, the changes observed in the objective function and constraints should be analyzed using parametric programming (Triantis & Oliver, 1998).

Wang and Liang argued that the objectives, constraints, and all coefficients should be fuzzified, enabling the solution of linear programming problems.

In general, Fuzzy Linear Programming provides faster and more flexible solutions in situations of uncertainty and lack of information, ensuring the most accurate decision-making.

Comparing the Zimmermann and Werners approaches, we can conclude the following conclusions:

- The Zimmermann approach finds a solution by combining multiple fuzzy constraints and objectives into a single total satisfaction function.
- The Werners approach establishes a hierarchy among fuzzy objectives and seeks flexible solutions. It offers more flexibility compared to Zimmermann's approach and may be more effective for multi-objective problems.
- These two approaches offer different solution strategies for decision problems involving uncertainty, providing a broad range of applications for fuzzy linear programming

2. Establishing a Fuzzy Linear Programming Model for the Turkish Cement Sector

In this section, the purpose, significance, limitations, and data collection process of the research are briefly discussed. The variables used in the model and the related data are presented in tables. Subsequently, the model related to the research is determined, and the results obtained within the model's scope are evaluated.

Table 1: Distances between regions (km).

	Ankara	Istanbul	Izmir	Antalya	Shanlıurfa	Van	Samsun
Ankara	0	449,9	585,3	483,5	842,5	1218,8	410,6
Istanbul	449,9	0	471,8	715	1289,9	1610,2	736,9
Izmir	585,3	471,8	0	455,2	1253,6	1776,3	1000,3
Antalya	483,5	715	455,2	0	966,2	1488,9	889,6
Shanlıurfa	842,5	1289,9	1253,6	966,2	0	543,3	839,5
Van	1218,8	1610,2	1776,3	1488,9	543,3	0	1010,8
Samsun	410,6	736,9	1000,3	889,6	839,5	1010,8	0

In an environment of uncertainties, this problem can be best addressed with a fuzzy linear programming model. The Werners approach was utilized in solving the problem. In this approach, although the membership functions of the constraints can be determined by the decision-maker, due to the fuzziness of the constraints, the membership function related to the objective function perceived as fuzzy cannot be predetermined by the decision-maker. In practice, the membership function related to the objective function was also not predetermined by the decision-maker. The following mathematical expression is used for representing fuzzy numbers.

In practice, when no information is provided by the decision-maker regarding the target of a fuzzy objective, the first step is to determine the values of Z^0 and Z^1. Here:

- Z^0 represents the minimum objective where no tolerance is applied, meaning a strict or non-flexible scenario.
- Z^1 represents the maximum objective where full tolerance is utilized, indicating a fully flexible scenario.

The values of Z^0 and Z^1 can also be expressed as the lower and upper limits of the objective function.

Production Amount Constraints: Businesses always operate with stock. Therefore, while producing, they need to predict how much production should be done based on their past experiences, as well as considering existing ready orders. Therefore, the supply and demand amounts of products are provided as fuzzy. Accordingly, the upper and lower limit values of the supply and demand for each business and distribution center are given in Table 3.

This table shows that although the amount of cement produced in Ankara is 6 689 781 tons, production can increase up to 9 243 155 tons if there is demand. In Istanbul factories, the amount of cement produced can increase from 7 629 819 tons to 10 432 620 tons. The amount of product produced in İzmir factories can vary between 2998271 and 3975294 tons, while the capacity in Şanlıurfa factories can range between 3736576 and 4178738 tons. The demand at the consumption centers is within the ranges specified in Table 3. When the values of the objective function are found at the lower and upper limits, the minimum Z^0 value of the objective function can be found first when the R/SIMPLEX software package is used at the lower limit values.

Table 2: Cement production capacity in Turkey in tons and percentages.

Regions	Capacity amount (Ton)	Capacity utilization Ratios (%)
Marmara	29.367.736	51,32
Aegean	14.333.611	32,78
Mediterranean	34.124.430	42,37
Black Sea	18.131.912	38,32
Central Anatolia	23.255.300	39,9
Eastern Anatolia	13.297.963	38,4
Southeastern Anatolia	11.241.408	38,84
Total	143.752.359	41,55

Table 3: Transportation model 2019.

	Ankara	Istanbul	Izmir	Antalya	Shanlıurfa	Van	Samsun	Supply
Ankara	0 Y11	449,9 Y12	585,3 Y13	483,5 Y14	842,5 Y15	1.218,8 Y16	410,6 Y17	6.689.781-9.243.155
Istanbul	449,9 Y21	0 Y22	471,8 Y23	715 Y24	1.289,9 Y25	1.610,2 Y26	736,9 Y27	7.629.819-10.432.620
Izmir	585,3 Y31	471,8 Y32	0 Y33	455,2 Y34	1.253,6 Y35	1.776,3 Y36	1.000,3 Y37	2.998.271-3.975.294
Antalya	483,5 Y41	715 Y42	455,2 Y43	0 Y44	966,2 Y45	1.488,9 Y46	889,6 Y47	5.281.085-7.288.883
Shanlıurfa	842,5 Y51	1.289,9 Y52	1.253,6 Y53	966,2 Y54	0 Y55	543,3 Y56	839,5 Y57	3.736.576-4.178.738
Van	1.218,8 Y61	1.610,2 Y62	1.776,3 Y63	1.488,9 Y64	543,3 Y65	0 Y66	1.010,8 Y67	3.012.990-4.985.013
Samsun	410,6 Y71	736,9 Y72	1.000,3 Y73	889,6 Y74	839,5 Y75	1.010,8 Y76	0 Y77	4.061.882 5.746.450
Demand	6.794.031-9.406.382	7.597.687-10.385.101	2.973.15-3.950.244	5.043.583-6.862.814	3.719.267-4.178.093	3.016.46 -4.955.688	4.045.836-5.673.795	

3. Solving The Problem Using The Werner's Method

The transportation information, transportation table, and mathematical form of the problem will be as follows. When the problem is solved with the R/SIMPLEX software package, the results are as follows:

Constraints: They are divided into two parts, supply, and demand.

Objective Function:

$$Z_{min} = \sum_{i=1}^{6}\sum_{j=1}^{6} C_{ij}Y_{ij} \text{ (i = production center),}$$

$$(j = \text{demand centers})$$

$$Z_{min} = 0 * Y_{11} + 449.9 * Y_{12} + 585.3 * Y_{13} + 483.5 * Y_{14} + 842.5 * Y_{15} + 1218.8 * Y_{16} + X_{17} * 410.6 + 449.9 * Y_{21} + 0 * Y_{22} + 471.8 * Y_{23} + 715 * Y_{24} + 1289.9 * Y_{25} + 1610.2 * Y_{26} + 736.9 * Y_{27} + 585.3 * X_{31} + 471.8 * X_{32} + 0 * X_{33} + 455.2 * X_{34} + 1253.6 * Y_{35} + 1776.3 * Y_{36} + 1000.3 * Y_{37} + 483.5 * Y_{41} + 715 * Y_{42} + 455.2 * Y_{43} + 0 * Y_{44} + 966.2 * Y_{45} + 1488.9 * Y_{46} + 889.6 * Y_{47} + 842.5 * Y_{51} + 1289.9 * Y_{52} + 1253.6 * Y_{53} + 966.2 * Y_{54} + 0 * Y_{55} + 543.3 * Y_{56} + 839.5 * Y_{57} + 1218.8 * Y_{61} + 1610.2 * Y_{62} + 1776.3 * Y_{63} + 1488.9 * Y_{64} + 543.3 * Y_{65} + 0 * Y_{66} + 1010.8 * Y_{67} + 410.6 * Y_{71} + 736.9 * Y_{72} + 1000.3 * Y_{73} + 889.6 * Y_{74} + 839.5 * Y_{75} + 1010.8 * Y_{76} + 0 * Y_{77}$$

Constraints: Divided into three categories: supply, demand, and positivity.

Supply Constraint:

$$\sum_{j=1}^{6} Y_{ij} \leq a_i \ \ (i - production\ center)$$

$$Y_{11} + Y_{12} + Y_{13} + Y_{14} + Y_{15} + Y_{16} + Y_{17} \leq \text{ 6.689.781;}$$

$$Y_{21} + Y_{22} + Y_{23} + Y_{24} + Y_{25} + Y_{26} + Y_{27} \leq \text{ 7.629.819;}$$

$$Y_{31} + Y_{32} + Y_{33} + Y34 + Y + Y_{36} + Y_{37} \leq \text{ 2.998.271;}$$

$$Y_{41} + Y_{42} + Y_{43} + Y_{44} + Y_{45} + Y_{46} + Y_{47} \leq \text{ 5.281.085}$$

$$Y_{51} + Y_{52} + Y_{53} + Y_{54} + Y_{55} + Y_{56} + Y_{57} \leq \ 3.736.576;$$

$$Y_{61} + Y_{62} + Y_{63} + Y_{64} + Y_{65} + Y_{66} + Y_{67} \leq \ 3.012.990;$$

$$Y_{71} + Y_{72} + Y_{73} + Y_{74} + X_{75} + Y_{76} + Y_{77} \leq \ 4.061.882;$$

Demand centers:

$$\sum_{i=1}^{6} Y_{ij} \geq \ b_j(j = demand\ centers)$$

$$Y_{11} + Y_{21} + Y_{31} + Y_{41} + Y_{51} + Y_{61} + Y_{71} \geq 6.794.031$$

$$Y_{12} + Y_{22} + Y_{32} + Y_{42} + Y_{52} + Y_{62} + Y_{72} \ \geq 7.597.687$$

$$Y_{13} + Y_{23} + Y_{33} + Y_{43} + Y_{53} + Y_{63} + Y_{73} \ \geq 2.973.153$$

$$Y_{14} + Y_{24} + Y_{34} + Y_{44} + Y_{54} + Y_{64} + Y_{74} \geq 5.043.583$$

$$Y_{15} + Y_{25} + Y_{35} + Y_{45} + Y_{55} + Y_{65} + Y_{75} \ \geq 3.719.267$$

$$Y_{16} + Y_{26} + Y_{36} + Y_{46} + Y_{56} + Y_{66} \ + Y_{76} \geq 3.016.468$$

$$Y_{17} + Y_{27} + Y_{37} + Y_{47} + Y_{57} + Y_{67} \ + Y_{77} \geq 4.045.836$$

The results of this application obtained using the R/SIMPLEX package program are shown in Table 4 and Table 5.

Table 4: Transportation table R/SIMPLEX solution results for the year 2019.

Variable	Value	Variable	Value	Variable	Value
Y11	**6.689.781**	Y34	0	Y57	0
Y12	0	Y35	0	Y61	0
Y13	0	Y36	0	Y62	0
Y14	0	Y37	0	Y63	0
Y15	0	Y41	**56.072**	Y64	0
Y16	0	Y42	0	Y65	**3.478**
Y17	0	Y43	0	Y66	**3.719.267**
Y21	**32.132**	Y44	**5.043.583**	Y67	0
Y22	**7.597.687**	Y45	0	Y71	**16.046**
Y23	0	Y46	0	Y72	0
Y24	0	Y47	0	Y73	0
Y25	0	Y51	0	Y74	0
Y26	0	Y52	0	Y75	0
Y27	0	Y53	0	Y76	0
Y31	0	Y54	0	Y77	**4.045.836**
Y32	0	Y55	**3.012.990**		
Y33	**2.973.153**	Y56	**0**		

Table 5: Transportation table for 2019.

	Ankara	Istanbul	Izmir	Antalya	Shanlıurfa	Van	Samsun	Supply
Ankara	0 6.689.781	449,9 0	585,3 0	483,5 0	842,5 0	1.218,8 0	410,6 0	6.689.781
Istanbul	449,9 32.132	0 7.597.687	471,8 0	715 0	1.289,9 0	1.610,2 0	736,9 0	7.629.819
Izmir	585,3 0	471,8 0	0 2.973.153	455,2 0	1.253,6 0	1.776,3 0	1.000,3 0	2.973153
Antalya	483,5 56.072	715 0	455,2 0	0 5.043,583	966,2 0	1.488,9 0	889,6 0	5.099.655
Shanlıurfa	842,5 0	1.289,9 0	1.253,6 0	966,2 0	0 3.012.990	543,3 0	839,5 0	3.012.990
Van	1.218,8 0	1.610,2 0	1.776,3 0	1.488,9 0	543,3 3.478	0 3.719.267	1.010,8 0	3.722.745
Samsun	410,6 16.046	736,9 0	1.000,3 0	889,6 0	839,5 0	1.010,8 0	0 4.045.836	4.061.882
Demand	6.794.031	7.597.687	2.973.153	5.043.583	3.016.468	3.719.267	4.045.836	

Total Cost= 0*6.689.781 + 449,9*32.132 + 0*7.597.687+ 2.973.153*0 + 483,5*56.072 + 0*3.012.990 + 543,3*3.478 + 0*3.719.267 + 0*3.719.267 + 0*3.012.990 + 410,6*16.046 + 0*4.045.836 = 50.045.083.8 (km)

Z^0=50.045.083.8 (km)

The upper limits for the supply and demand constraints are expressed as follows:

Supply Constraint:

$$\sum_{j=1}^{6} Y_{ij} \leq a_i \ (i - Production\ Center)$$

$$Y_{11} + Y_{12} + Y_{13} + Y_{14} + Y_{15} + Y_{16} + Y_{17} \leq \ 9.243.155;$$

$$Y_{21} + Y_{22} + Y_{23} + Y_{24} + Y_{25} + Y_{26} + Y_{27} \leq \ 10.432.620;$$

$$Y_{31} + Y_{32} + Y_{33} + Y_{34} + Y_{35} + Y_{36} + Y_{37} \leq \ 3.975.294;$$

$$Y_{41} + Y_{42} + Y_{43} + Y_{44} + Y_{45} + Y_{46} + Y_{47} \leq \ 7.288.883$$

$$Y_{51} + Y_{52} + Y_{53} + Y_{54} + Y_{55} + Y_{56} + Y_{57} \leq 4.178.738;$$

$$Y_{61} + Y_{62} + Y_{63} + Y_{64} + Y_{65} + Y_{66} + Y_{67} \leq 4.985.013;$$

$$Y_{71} + Y_{72} + Y_{73} + Y_{74} + Y_{75} + Y_{76} + Y_{77} \leq \ 5.746.450;$$

Demand Constraint:

$$\sum_{i=1}^{6} Y_{ij} \geq \ \ b_j (j = demand\ centers)$$

$$Y_{11} + Y_{21} + Y_{31} + Y_{41} + Y_{51} + Y_{61} + Y_{71} \geq 9.406.382$$

$$Y_{12} + Y_{22} + Y_{32} + Y_{42} + Y_{52} + Y_{62} + Y_{72} \ \geq 10.385.101$$

$$Y_{13} + Y_{23} + Y_{33} + Y_{43} + Y_{53} + Y_{63} + Y_{73} \ \geq 3.950.244$$

$$Y_{14} + Y_{24} + Y_{34} + Y_{44} + Y_{54} + Y_{64} + Y_{74} \geq 6.862.814$$

$$Y_{15} + Y_{25} + Y_{35} + Y_{45} + Y_{55} + Y_{65} + Y_{75} \ \geq 4.178.093$$

$$Y_{16} + Y_{26} + Y_{36} + Y_{46} + Y_{56} + Y_{66} \ + Y_{76} \geq 4.955.688$$

$$Y_{17} + Y_{27} + Y_{37} + Y_{47} + Y_{57} + Y_{67} \ + Y_{77} \geq 5.673.795$$

R/Simplex solution results are provided in Table 6 and Table 7.

Table 6: Transportation table R/Simplex solution results for 2019 upper bound.

Variable	Value	Variable	Value	Variable	Value
Y11	**9.243.155**	Y34	0	Y57	0
Y12	0	Y35	0	Y61	0
Y13	0	Y36	0	Y62	0
Y14	0	Y37	0	Y63	0
Y15	0	Y41	**43.053**	Y64	0
Y16	0	Y42	0	Y65	
Y17	0	Y43	0	Y66	**4.955.688**
Y21	**47.519**	Y44	**6.862.814**	Y67	0
Y22	**10.385.101**	Y45	0	Y71	**72.655**
Y23	0	Y46	0	Y72	0
Y24	0	Y47	0	Y73	0
Y25	0	Y51	0	Y74	0
Y26	0	Y52	0	Y75	0
Y27	0	Y53	0	Y76	0
Y31	0	Y54	0	Y77	**5.673.795**
Y32	0	Y55	**4.178.093**		
Y33	**3.950.244**	Y56	**0**		

Table 7: Transportation table for 2019 upper bound.

	Ankara	İstanbul	İzmir	Antalya	Şanlıurfa	Van	Samsun	Supply
Ankara	**0** **9.243.155**	**449,9** **0**	**585,3** **0**	**483,5** **0**	**842,5** **0**	**1.218,8** **0**	**410,6** **0**	9.243.155
Istanbul	**449,9** **47.519**	**0** **10.385.101**	**471,8** **0**	**715** **0**	**1.289,9** **0**	**1.610,2** **0**	**736,9** **0**	10.432.62
Izmir	**585,3** **0**	**471,8** **0**	**0** **3.950.244**	**455,2** **0**	**1.253,6** **0**	**1.776,3** **0**	**1.000,3** **0**	3.950.244
Antalya	**483,5** **43.053**	**715** **0**	**455,2** **0**	**0** **6.862.814**	**966,2** **0**	**1.488,9** **0**	**889,6** **0**	6.905.867
Şanlıurfa	**842,5** **0**	**1.289,9** **0**	**1.253,6** **0**	**966,2** **0**	**0** **4.178.093**	**543,3** **0**	**839,5** **0**	4.178.093
Van	**1.218,8** **0**	**1.610,2** **0**	**1.776,3** **0**	**1.488,9** **0**	**543,3** **0**	**0** **4.955.688**	**1.010,8** **0**	4.955.688
Samsun	**410,6** **72.655**	**736,9** **0**	**1.000,3** **0**	**889,6** **0**	**839,5** **0**	**1.010,8** **0**	**0** **5.673.795**	5.746.450
Demand	9.406.382	10.385.101	3.950.244	6.862.814	4.178.093	4.955.688	5.673.795	

According to Table 7, the total transportation cost is Z=72.027.066,6 (km), which means the upper bound of the objective function is

Z^1=72.027.066,6(km)

Now, let's write the membership function of the objective function:

$$\mu_0 = \begin{cases} 1; & \text{if } c^T x > 72.027.066,6 \\ 1 - \dfrac{72.027.066,6 - c^T x}{21.981.982,8}; & \text{if } 50.045.083,8 < c^T x < 72.027.066,6 \\ 0; & \text{if } c^T x < 50.045.083,8 \end{cases}$$

Here: Z^0=50.045083,8, Z^1=72.027.066,6 .

The difference is $p_0 = Z^1 - Z^0$= 21.981.982,8

Membership functions of constraints can be written as follows:

$$\mu_{i1(x)} = \begin{cases} 1; & \text{if } (Ax)_i < b_i \\ 1 - \dfrac{(Ax)_i - b_i}{p_i}; & \text{if } b_i \le (Ax)_i \le b_i + p_i \\ 0; & \text{if } (Ax)_i > b_i + p_i \end{cases}$$

$$\mu_{i2(x)} = \begin{cases} 1; & \text{if } (\mathrm{Bx})_i > c_i + p_i \\ 1 - \dfrac{(\mathrm{Bx})_i - c_i}{p_i}; & \text{if } c_i \le (\mathrm{Bx})_i \le c_i + p_i \\ 0; & \text{if } (\mathrm{Bx})_i < c_i \end{cases}$$

As specified in Section 1.7, Werners' approach is a symmetric method and a fuzzy linear programming model that ensures the satisfaction of both the objective function and the constraints simultaneously. To convert Werners' model into a classical linear programming model, let us use the variable λ.

$C^T X \ge b_0 - (1-\lambda)\, p_0$

$(AX)_i \le b_i + (1-\lambda)\, p_i$

$(BX)_i \ge c_i + (1-\lambda)\, p_i$

$X_i \ge 0$

To solve the fuzzy linear programming problem similarly to a classical linear programming model, the constants on the right-hand side are kept unchanged. Finally, it can be reformulated as follows

0*Y11+449.9*Y12+585.3*Y13+483.5*Y14+842.5*Y15+1218.8*Y16+410.6*Y17

+449.9*Y21+0*Y22+471.8*Y23+715*Y24+1289.9*Y25+1610.2*Y26+736.9*Y27

+585.3*Y31+471.8*Y32+0*Y33+455.2*Y34+1253.6*Y35+17776.3*Y36+

1000.3*Y37+48.5Y41+715*Y42+455.2*Y43+0*Y44+966.2*Y45+1488.9*Y46+

889.6*Y47+842.5*Y51+1289*Y52+1253.6*Y62+1776.3*Y63+1488.9*Y64+

543.3*Y65+0*Y66+1010.8*Y67+410.6*Y71+736.9Y72+1000.3*Y73+889.6*,Y74+

839.5*Y75+1010.8*Y76+0*Y77-21.981.982,8λ≥ 50.045.083,8

For the supply and demand constraints:

$Y_{11} + Y_{12} + Y_{13} + Y_{14} + Y_{15} + Y_{16} + Y_{17} + 2.553.374\lambda \leq$ 9.243.155;

$Y_{21} + Y_{22} + Y_{23} + Y_{24} + Y_{25} + Y_{26} + Y_{27} + 2.802.801\lambda \leq$ 10.432.620 ;

$Y_{31} + Y_{32} + Y_{33} + Y_{34} + Y_{35} + Y_{36} + Y_{37} + 977.023\lambda \leq$ 3.975.294;

$$Y_{41} + Y_{42} + Y_{43} + Y_{44} + Y_{45} + Y_{46} + Y_{47} + 2.007.798\lambda \leq 7.288.883$$

$$Y_{51} + Y_{52} + Y_{53} + Y_{54} + Y_{55} + Y_{56} + Y_{57} + 1.165.748\lambda \leq 4.178.738;$$

$$Y_{61} + Y_{62} + Y_{63} + Y_{64} + Y_{65} + Y_{66} + Y_{67} + 1.248.437\lambda \leq 4.985.013;$$

$$Y_{71} + Y_{72} + Y_{73} + Y_{74} + Y_{75} + Y_{76} + Y_{77} + 1.684.568\lambda \leq 5.746.450;$$

$$Y_{11} + Y_{21} + Y_{31} + Y_{41} + Y_{51} + Y_{61} + Y_{71} + 2.611.626\lambda \geq 9.406.382$$

$$Y_{12} + Y_{22} + Y_{32} + Y_{42} + Y_{52} + Y_{62} + Y_{72} + 2.787.414\lambda \geq 10.385.101$$

$$X_{13} + Y_{23} + Y_{33} + Y_{43} + Y_{53} + Y_{63} + Y_{73} + 977.091\lambda \geq 3.950.244$$

$$Y_{14} + Y_{24} + Y_{34} + Y_{44} + Y_{54} + Y_{64} + Y_{74} + 1.819.231\lambda \geq 6.862.814$$

$$Y_{15} + Y_{25} + Y_{35} + Y_{45} + Y_{55} + Y_{65} + Y_{75} + 1.161625\lambda \geq 4.178.093$$

$$Y_{16} + Y_{26} + Y_{36} + Y_{46} + Y_{56} + Y_{66} + Y_{76} + 1.236.421\lambda \geq 4.955.688$$

$$Y_{17} + Y_{27} + Y_{37} + Y_{47} + Y_{57} + Y_{67} + Y_{77} + 1.627.959\lambda \geq 5.673.795$$

According to Werner's approach, in order to find the optimal solution, the α-section set of the solution space with equal membership degrees should be considered as a fuzzy decision set. In the equal membership solution space, λ is set to 0.5. Accordingly, the model takes the following form.

0*Y11+449.9*Y12+585.3*Y13+483.5*Y14+842.5*Y15+1218.8*Y16+410.6*Y17

+449.9*Y21+0*Y22+471.8*Y23+715*Y24+1289.9*Y25+1610.2*Y26+736.9*Y27

+585.3*Y31+471.8*Y32+0*Y33+455.2*Y34+1253.6*Y35+17776.3*Y36+

1000.3*Y37+48.5Y41+715*Y42+455.2*Y43+0*Y44+966.2*Y45+1488.9*Y46+

889.6*Y47+842.5*Y51+1289*Y52+1253.6*Y62+1776.3*Y63+1488.9*Y64+

543.3*Y65+0*Y66+1010.8*Y67+410.6*Y71+736.9Y72+1000.3*Y73+889.6*Y74+

839.5*Y75+1010.8*Y76+0*Y77 ≥ 61.036.075,35

When considering the value of λ in the final system of inequalities, the supply and demand constraints take the following form:

$$Y_{11} + Y_{12} + Y_{13} + Y_{14} + Y_{15} + Y_{16} + Y_{17} \leq 7.966.468$$

$$Y_{21} + Y_{22} + Y_{23} + Y_{24} + Y_{25} + Y_{26} + Y_{27} \leq 9.031.219{,}5$$

$$Y_{31} + Y_{32} + Y_{33} + Y_{34} + Y_{35} + Y_{36} + Y_{37} \leq \ 3.486.782{,}5$$

$$Y_{41} + Y_{42} + Y_{43} + Y_{44} + Y_{45} + Y_{46} + Y_{47} \leq 6.284.984$$

$$Y_{51} + Y_{52} + Y_{53} + Y_{54} + Y_{55} + Y_{56} + Y_{57} \leq 3.595.864$$

$$Y_{61} + Y_{62} + Y_{63} + Y_{64} + Y_{65} + Y_{66} + Y_{67} \leq \ 4.360.794{,}5$$

$$Y_{71} + Y_{72} + Y_{73} + Y_{74} + Y_{75} + Y_{76} + Y_{77} \leq 4.904.166$$

$$Y_{11} + Y_{21} + Y_{31} + Y_{41} + Y_{51} + Y_{61} + Y_{71} \geq 8.100.568{,}9$$

$$Y_{12} + Y_{22} + Y_{32} + Y_{42} + Y_{52} + Y_{62} + Y_{72} \geq 8.991.394$$

$$Y_{13} + Y_{23} + Y_{33} + Y_{43} + Y_{53} + Y_{63} +, Y_{73} \geq 3.461.698{,}5$$

$$Y_{14} + Y_{24} + Y_{34} + Y_{44} + Y_{54} + Y_{64} + Y_{74} \geq 5.953.198{,}5$$

$$Y_{15} + Y_{25} + Y_{35} + Y_{45} + Y_{55} + Y_{65} + Y_{75} \geq 3.597.280{,}5$$

$$Y_{16} + Y_{26} + Y_{36} + Y_{46} + Y_{56} + Y_{66} \ + Y_{76} \geq 4.337.477{,}5$$

$$Y_{17} + Y_{27} + Y_{37} + Y_{47} + Y_{57} + X_{67} \ + Y_{77} \geq 4.859.815{,}5.$$

The results obtained with this application using the R/SIMPLEX package program are shown in Tables 8 and 9.

Table 8: Transportation table R/SIMPLEX solution results.

Variable	Value	Variable	Value	Variable	Value
Y11	**7.966.468**	Y34	0	Y57	0
Y12	0	Y35	0	Y61	0
Y13	0	Y36	0	Y62	0
Y14	0	Y37	0	Y63	0
Y15	0	Y41	**49.924,9**	Y64	0
Y16	0	Y42	0	Y65	**1.416,5**
Y17	0	Y43	0	Y66	**4.337.477,5**
Y21	**39.825,5**	Y44	**5.953.198,5**	Y67	0
Y22	**8.991.394**	Y45	0	Y71	**44.350,5**
Y23	0	Y46	0	Y72	0
Y24	0	Y47	0	Y73	0
Y25	0	Y51	0	Y74	0
Y26	0	Y52	0	Y75	0
Y27	0	Y53	0	Y76	0
Y31	0	Y54	0	Y77	**4.859.815,5**
Y32	0	Y55	**3.595.864**		
Y33	**3.461.698,5**	Y56	**0**		

Table 9: Transportation model 2019.

	Ankara	Istanbul	Izmir	Antalya	Shanlıurfa	Van	Samsun	Supply
Ankara	0 7.966.468	449,9 0	585,3 0	483,5 0	842,5 0	1.218,8 0	410,6 0	7.966.468
İstanbul	449,9 39.825,5	0 8.991.394	471,8 0	715 0	1.289,9 0	1.610,2 0	736,9 0	9.031.219,5
Izmir	585,3 0	471,8 0	0 3.461.698,5	455,2 0	1.253,6 0	1.776,3 0	1.000,3 0	3.486.782,5
Antalya	483,5 49.924,9	715 0	455,2 0	0 5.953.198,5	966,2 0	1.488,9 0	889,6 0	6.003.123.4
Shanlıurfa	842,5 0	1.289,9 0	1.253,6 0	966,2 0	0 3.595.864	543,3 0	839,5 0	3.595.864
Van	1.218,8 0	1.610,2 0	1.776,3 0	1.488,9 0	543,3 1.416,5	0 4.337.477,5	1.010,8 0	4.338.894
Samsun	410,6 44.350,5	736,9 0	1.000,3 0	889,6 0	839,5 0	1.010,8 0	0 4.859.815,5	4.904.166
Demand	8.100.568,9	8.991.394	3.461.698,5	5.953.198,5	3.597.280,5	4.337.477,5	4.859.815,5	

Z=449,9*39.825,5+483,5*49.924,9+543,3*1.416,5+410,6* 44.350,5=61.036.081.35

4. Conclusions

Today's manufacturers operate in a highly competitive environment with the goal of maximising profits. One of the most significant costs affecting profit maximisation is the cost of transportation. Manufacturers need an optimal distribution plan to control transportation costs. On the other hand, complexity, uncertainty, and lack of information are prevalent in today's world. When these uncertainties in decision-making processes are tried to be solved by classical logic, objective results cannot be obtained. In such cases, the use of fuzzy linear programming, which combines fuzzy set theory introduced by Zadeh in 1965 with linear programming, is suitable.

In this context, the cement industry, where transportation costs are high, is addressed in this section. Samples were taken from seven different regions of Turkey, and the most optimal distribution plan and objective function were determined in an uncertain environment using a fuzzy linear programming model.

The study focuses on the optimal distribution problem of cement produced in factories operating in the Turkish Cement Industry to demand points. When creating the transportation model, each region's geographical center was taken as a city, and supply and demand points were identified as geographical regions. The regions are represented by Istanbul for the Marmara Region, Izmir for the Aegean Region, Antalya for the Mediterranean Region, Ankara for the Central Anatolia Region, Samsun for the Black Sea Region, Van for the Eastern Anatolia Region, and Shanlıurfa for the Southeastern Anatolia Region.

The part1 of this chapter of this study focuses on the emergence of fuzzy logic and fuzzy linear programming, including the Zimmermann and Werners approaches. The part2 of chapter studies optimal distribution with minimum cost in the Cement Sector in Turkey using the Werners approach.

The transportation problem has been mathematically modeled within the scope of fuzzy programming. The R/SIMPLEX package program was used for solving the problem. Based on the solutions obtained within the framework of fuzzy programming with the Werners approach, a minimum-cost optimal distribution plan has been proposed according to the existing supply and demand.

In this study no tolerance was given to the objective function for the Turkish cement sector where the application was performed; only tolerance values for constraints were provided. Therefore, Werners' approach, among linear programming solution approaches, was used in solving the model created based on business data. According to Werners' approach, to reach the optimal solution, the α-cut set of the solution space with equal membership degrees must be considered as the fuzzy decision set. Accordingly, the optimal solution values correspond to the value where α is 0.5. Considering this situation, the lower and upper limits of the Objective Function, i.e., minimum and maximum values, were obtained, and the optimal value of the Objective Function and the optimal distribution plan to consumption centers were found.

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